The Crafoord Prize 2008

This year's Crafoord Prize combines abstract mathematics with astrophysics. It is being awarded for mathematical discoveries that are significant for the fundamental laws of nature and for research on black holes and the early Universe. This year's laureates have each made decisive contributions to our understanding of the origin of the Universe.

Mathematics

This year's laureates in mathematics, the mathematician **Maxim Kontsevich** and the theoretical physicist **Edward Witten**, have used the methodology of physics to develop a new mathematics. Their results have great importance for fundamental physics such as particle physics and string theory.

String mathematics

Throughout its history physics has developed in intimate interaction with mathematics. It is common for the mathematics devised by inquisitive mathematicians with no thought of any application to turn out to be unexpectedly useful in physics. But sometimes the opposite also occurs. Methods intended for use with physical problems turn out to pave the way to new mathematics. This is true to a very high degree of the research to which this year's laureates Maxim Kontsevich and Edward Witten have devoted their endeavours.

Witten is one of the most eminent theoretical physicists of all time and has devoted his research mainly to string theory. String theory attempts to unite quantum mechanics with the general relativity theory for gravitation to form a whole without contradictions. Creating a theory that would in this way describe all four fundamental natural forces would be a major breakthrough.

Figure 1.
According to string theory, the particles of which our world is formed are different manifestations of strings. A string of this kind will behave as an electron, quark, photon or some other particle, depending on how it vibrates.

String theory implies that space must have more dimensions than the three we recognize in our daily lives. Six or even seven extra ones are needed if the equations are to be resolved. The extra dimensions must be incredibly small to have avoided discovery in the successful experiments that have been undertaken up to now in particle accelerators all over the world. But string theory predicts that with enough energy it will finally be possible to discern additional directions in space.

To calculate these theoretical predictions a new mathematics is needed. This is exactly what Witten has established and with the help of methods borrowed from theoretical physics he has performed calculations that were previously impossible. Kontsevich in his turn has used the same kind of physical intuitions but has made yet another pioneering advance to verify that the methods inspired by physics do in fact function mathematically and yield correct results. In his work Kontsevich has shown that he is one of the most original mathematicians of our day.

Doughnuts, knots and fishing nets

One important area of mathematics involves the classification and enumeration of various geometrical objects. A simple example can be found in closed surfaces. If you are not permitted to tear them apart or patch them together but may bend and stretch them any way you want, they can be classified in terms of the number of holes. A bun has no holes and a doughnut generally one hole.

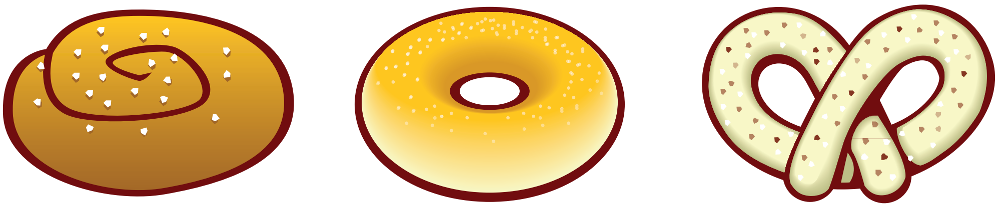

Figure 2.
One way of distinguishing between different forms of bun is to count the number of holes. In the same way mathematicians differentiate between kinds of geometrical object.

When there are more dimensions than two it becomes more difficult to keep track of geometry and there is a great deal of very exciting mathematics still to be discovered. One particularly significant and beautiful characteristic studied by Kontsevich and Witten is what is known as mirror symmetry. This is related to how the extra dimensions of strings may in certain circumstances appear totally different geometrically but still produce the same physics in ordinary four-dimensional space-time comprising height, width, depth and time.

The methods on which these mathematical advances are based have been borrowed from particle physics, where the powerful quantum field theory is used to describe elementary particles and their variations. By adding together all the ways in which particles can move and metamorphose into new ones it is possible to use quantum field theory to calculate the probabilities of different results from the collisions of particles in particle accelerators.

These sums can be expressed with the help of a *Feynman diagram*. This diagram has been named after the American physicist and Nobel Laureate Richard Feynman and displays the various ways in which particles are able to move. Physicists have developed methods for calculating these possible sums and they have also been used successfully.

There is often amazing agreement with the experimental results – the calculations really do work.

Despite the success there still remain difficulties in principle with the methods. Often disturbing infinities occur that should really make the results meaningless. The general opinion among physicists is that it has been possible to overcome these problems and that the results obtained are both meaningful and correct. But from the point of view of a mathematician this is not entirely satisfactory. Can calculations really be made like this? Is everything really well defined? If a result is to be described as mathematically certain, stringent requirements apply. What underlies the success of this year's laureates is that despite these misgivings they have dared to use the methods of physicists – in particular the Feynman diagram – in the search for new mathematics.

One example concerns gravitation in two dimensions, which in principle involves adding together all the different ways in which a surface can be knobbly. Witten was able to intuit a relationship between two distinct ways of solving the problem, where one method consisted of replacing the geometrical surface with a fishing net of the same shape composed of Feynman diagrams. By using the Feynman diagrams in a totally new way Kontsevich was able to show that Witten's conjectures were totally correct.

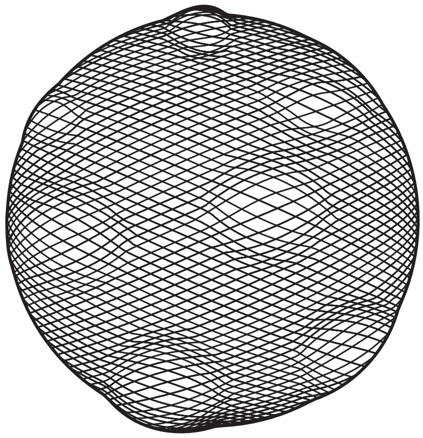

Figure 3.
By using a net constructed of Feynman diagrams it is possible to create surfaces with different shapes and analyze their characteristics, for example in what way they are knobbly.

Something as seemingly simple as a knot also conceals very exciting mathematics in which similar methods can be used. By envisaging the rope in the knot as the path of a particle Witten was able to devise a mathematical expression that distinguishes between different kinds of knots. Kontsevich could take this further to show that the mathematics really makes sense.

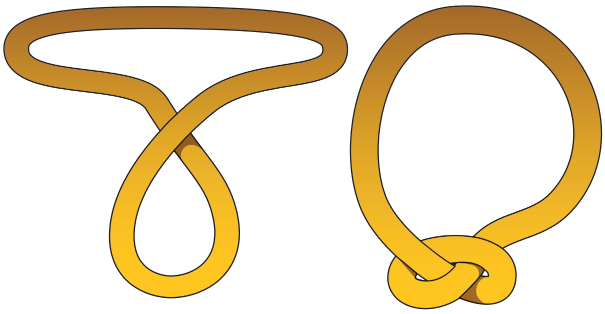

Figure 4.
Differentiating in mathematical terms between various types of knots is a difficult problem for which the laureates have found successful methods.

The mathematics developed by Witten and Kontsevich inspired by fundamental physics is of major principal interest. The results do not depend on how and when it will be possible to test string theory and it is more than likely that it will find its application in totally different areas.

Many physicists claim, however, that with the help of cosmology, which studies the origins of the Universe together with its development and large-scale structure, it will be possible to search for clues to the new worlds in which all this mathematics will play a decisive role. This takes us on to the second half of this year's Crafoord Prize.

Astronomy

This year's laureate in astronomy, **Rashid Sunyaev**, has devoted his research to the study of the most extreme processes in the Universe. He has developed theoretical models for the cosmological background radiation and how black holes devour matter. He has also been the leader of research teams working with instruments in satellites and space stations.

How to see a black hole

A black hole describes a region in space where gravitation is so strong that nothing, not even light, can escape from it. The first speculations about objects of this kind date back to the 18th century, but it was not until help was provided by Einstein's general relativity theory that it was possible to understand what was happening.

Black holes can form when a giant star dies in the form of a supernova. The centre of the star collapses, concentrating an enormous amount of matter in a small area. Black holes of this kind can weigh up to ten or twenty times more than the sun while their radii can be calculated in kilometres.

We also know that there are considerably larger black holes at the core of most galaxies. These black holes can weigh several million or billion times more than our sun does and be of the same size as our solar system. Our galaxy, the Milky Way, conceals a large black hole at its centre.

Close up, a black hole would look like a large and totally black sphere – not even light can escape from it. It is therefore reasonable to conclude that black holes must remain difficult if not impossible to discover. But paradoxically, black holes are among the most powerful sources of radiation in the entire Universe. The theory behind how this comes about has been worked out by Sunyaev together with Russian astrophysicist Nikolay Shakura. Their work is one of the most often cited sources in modern astrophysics.

The secret behind the visibility of black holes is that they are often close to stars or other forms of matter. If the star that exploded as a supernova was not a solitary but part of a double star system – and the system survived the explosion – the black hole can attract matter from its buffeted partner. In the same way, stray stars can become the victims of the enormous black holes at the core of galaxies.

The matter that tumbles into the black hole is not devoured immediately but first goes into orbit. This rapidly rotating disc of matter is called the *accretion disc,* and its structure and characteristics are described in the theory developed by Sunyaev and Shakura.

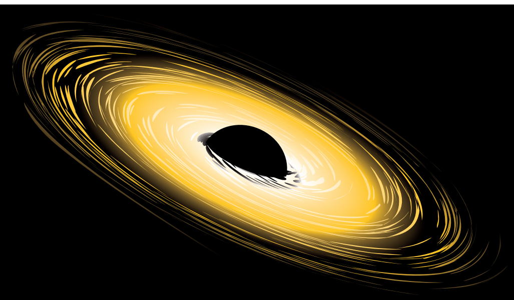

Figure 5.
Matter that falls into a black hole forms a thin, rapidly rotating disc. Large black holes surrounded by discs of this kind are concealed at the cores of galaxies.

When the clouds of matter in the disc rub against each other they are heated by friction. This friction also has the effect that they get closer and closer to the black hole. Finally they tumble into the black hole but before doing so they are able to emit x-ray radiation that is powerful enough to be measured from Earth. We cannot therefore observe the black hole directly but can still clearly prove its existence through the radiation of the accretion disc. This is how it has been known for some decades that black holes really exist.

Sunyaev's and Shakura's theory about accretion discs can be used for both small black holes created by exploding supernovae and also for the large black holes at galactic cores. The most luminous objects in the entire Universe – quasars – can be explained in this way as active galactic cores with black holes as their energy source.

Light from Big Bang

Sunyaev has also contributed to cosmology and our possibilities of finding out what happened when the Universe was created. Together with another eminent Russian astrophysicist, Yakov B. Zeldovich, Sunyaev laid the groundwork for today's successful attempts to determine the characteristics of the Universe using the cosmological background radiation.

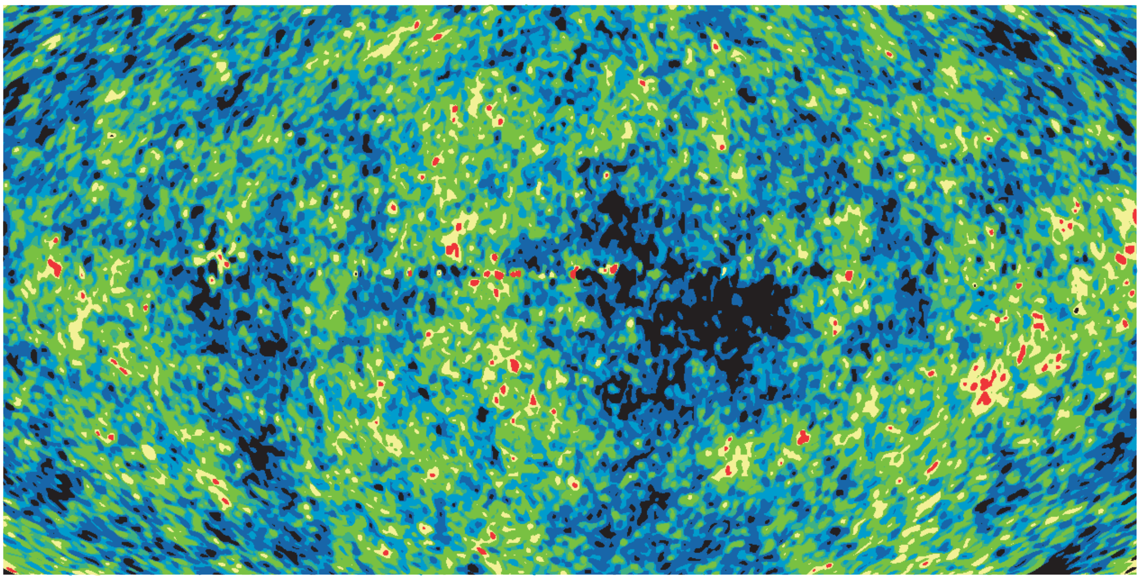

Figure 6.
The distribution of the cosmological background radiation in space can be determined using a telescope that is sensitive to microwaves. The structure of the radiation offers clues about what happened during Big Bang. Image: NASA/WMAP Science Team, http://map.gsfc.nasa.gov

This radiation derives from a period a few hundred thousand years after Big Bang, when the Universe became transparent for the first time. The theory about how this took place forms an important part of Sunyaev's and Zeldovich's work. Since then the radiation has travelled through the Universe with virtually no interruption and can now be observed in the form of microwaves. When we study the cosmological background radiation we are looking almost 14 billion years back in time.

This radiation is not, however, totally uniform but contains important clues about what happened in the early Universe. Enormous waves of sound from Big Bang swept through the hot matter and in their turn gave rise to variations in the temperature of the background radiation that we can now observe. By examining the degree of variation in temperature on different scales we can analyse the progress of the sound waves and this enables us to make deductions about the characteristics of the Universe. The impact of the sound waves on background radiation was predicted by both Sunyaev/Zeldovich and P. J. E. Peebles (Crafoord Laureate in 2005)/J.T. Yu independently of each other as early as 1970, and their calculations have been verified with the help of observations made from satellites and balloons. The study of the structure of the cosmological background radiation is one of the absolutely most important methods of attaining knowledge about the early history of the Universe.

The background radiation can also be used to find out how the matter of the Universe was distributed a long time after Big Bang. During the billions of years it has taken to reach us, the light particles or photons in the background radiation have traversed galaxies and clusters of galaxies. The photons can be affected by the clusters of galaxies, for instance through collisions with the electrons in the hot clouds in which many of these clusters are embedded. This effect is called the Sunyaev-Zeldovich effect and combined with other measurements, of x-ray radiation in particular, it can offer vital clues about the features of our Universe. In this way help can be found in measuring distances to clusters of galaxies and more information can be provided about dark matter and dark energy, which are assumed to form a major component of the Universe but about which we still do not know a great deal.

Figure 7.
Image captured by the Hubble Space Telescope of an enormous cluster of galaxies 9 billion light years away from Earth. With the help of the Sunyaev-Zeldovich effect the next generation of radio telescopes will be able to detect clusters of galaxies considerably further away, at the very boundaries of the visible Universe. Unlike radiation in the visible wavelengths, as in this image, the Sunyaev-Zeldovich signal does not become weaker because of distance. Image: Hubble Space Telescope, NASA, ESA, J. Blakeslee (JHU), M. Postman (STScI) and P. Rosati (ESO), http://hubblesite.org

Sunyaev has combined theory and observation as few others have. The observations of the cosmological background radiation and high-energy radiation from the cosmos that Sunyaev has pioneered constitute one of the absolutely most important and active areas in modern astronomy. Sunyaev continues to be a leading figure in these areas.

LINKS AND FURTHER READING

Articles:

"*A Universe of Disks*" (2004) by Omer Blaes, Scientific American Magazine, October 2004, 8 pp.

"*Cosmic microwave background radiation*" at Science Daily. Read the full article and watch graphics at: *www.sciencedaily.com/articles/c/cosmic_microwave_background_radiation.htm*

"*The Cosmic Symphony*" (2004) by Wayne Hu and Martin White, Scientific American Magazine, February 2004, 10 pp.

Illustrations: Typoform

Books:

"*The Elegant Universe – Superstrings, Hidden Dimensions, and the Quest for the Ultimate Theory*" (1999) by Brian Greene, W.W. Norton & Company, 464 pp.

"*Gravity's Fatal Attraction: Black Holes in the Universe*" (1998) by M. Begelman and M. J. Rees., W. H. Freeman and Company, 246 pp.

"*Enumerative Geometry and String Theory*" (2006) by Sheldon Katz, American Mathematical Society, Institute for Advanced Study, Student Mathematical Library, Vol. 32, 206 pp.

Links:

Many links to articles at Edward Witten's homepage: *http://www.sns.ias.edu/~witten/*

Search for "Knot theory" at Wikipedia. Lots of images.

WMAP *(see figure 6 page 5): http://map.gsfc.nasa.gov*

Black holes: *http://hubblesite.org/explore_astronomy/black_holes/index.html*

and *http://hubblesite.org/explore_astronomy/black_holes/modules.html*

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