The Rolf Schock Prize Symposium in Mathematics

Date: 16-17 October 2018
Venue: Institut Mittag-Leffler, Auravägen 17, Djursholm, Sweden

Tuesday 16 October

13.15 Three problems on trigonometric sums
Yves Meyer, École normale supérieure Paris-Saclay, France

14.00 Short break

14.15 Harmonic measure for higher dimensional boundaries
Guy David, Université Paris-Sud, France

15.15 Coffee break

15.30 Harmonic Analytic Geometry on subsets of ”Empirical models”
Ronald Coifman, Rolf Schock Laureate in Mathematics 2018, Yale University, USA

16.15 Short break

16.30 Learning and Geometry for Stochastic Dynamical Systems in high dimensions: Diffusions on Manifold and Agent-based systems
Mauro Maggioni, Johns Hopkins University, USA

17.15 Short break

17.30 Deep Representations with Applications in Cryo-Electron Microscopy and Data Science
Roy R. Lederman, Yale University, USA

Wednesday 17 October

09.15 Progress on singular Brascamp-Lieb inequalities
Christophe Thiele, Hausdorff Center for Mathematics, Germany

10.00 Coffee break

10.30 From Wavelets to Diffusion geometry - reflections of some efforts handling large data sets
Jan-Olov Strömberg, KTH Royal Institute of Technology, Sweden

11.15 Short break

11.30 Graphs, Eigenfunctions, and Metrics in Artificial Intelligence
Peter W. Jones, Yale University, USA

12.15 End of symposium

The symposium is free of charge and open to the public but registration is required for all participants. Registration and more information at www.kva.se/SchockMathematics2018

Abstracts

Three problems on trigonometric sums
Yves Meyer, École normale supérieure Paris-Saclay, France

Let Λ be a uniformly discrete set of points in $\R^n.$ When do local estimates on trigonometric sums with frequencies in Λ imply global estimates? This problem has a long history when the L^2 norm is used. New results are given here when the norm is a weighted L^{∞} norm.

Harmonic measure for higher dimensional boundaries
Guy David, Université Paris-Sud, France

It is tempting to define a notion of harmonic measure for open sets of the Euclidean space that are bounded by sets E of dimension less than $n-1$ (we'll take them Ahlfors-regular). Since the usual Brownian paths don't always see such a small set E, we are led to considering degenerate elliptic operators, with coefficients that tend to infinity, in a controlled way, at the boundary. For these we can solve Dirichlet problems, define a harmonic measure, and start to wonder how the regularity of that measure are related to the geometry of E. For instance, one can prove an analogue of Dahlberg's theorem in this context. The lecture describes joint work with M. Engelstein, J. Feneuil, and S.Mayboroda.

Harmonic Analytic Geometry on subsets of ”Empirical models”
Ronald Coifman, Rolf Schock Laureate in Mathematics 2018, Yale University, USA

Our goal is to describe a recent evolution of Harmonic Analysis to generate analytic tools for the joint geometric organization of the of subsets of R^n with the analysis of functions and operators restricted to these subsets. In this analysis we establish a duality between the geometry of functions and the geometry of the space. The methods are used to automate various analytic organizations, as well as to enable informative data analysis. These tools extend to higher order tensors, to combine dynamic analysis of changing structures.
In particular we view these tools as necessary to enable automated empirical modeling, in which the goal is to model dynamics in nature, {ab initio},through observations alone.

Learning and Geometry for Stochastic Dynamical Systems in high dimensions: Diffusions on Manifold and Agent-based systems
Mauro Maggioni, Johns Hopkins University, USA

We discuss geometry-based statistical learning techniques for performing model reduction and modeling of certain classes of stochastic high-dimensional dynamical systems. In the first scenario, we consider the case of systems that are well-approximated by diffusions on low-dimensional manifold. Neither the process nor the manifold are known: we assume we only have access to an (expensive) simulator that can return short paths of the stochastic system, given an initial condition.

We introduce a statistical learning framework for estimating local approximations to the system, then pieced together to form a fast global reduced model for the system, called ATLAS. ATLAS is guaranteed to be accurate (in the sense of producing stochastic paths whose distribution is close to that of paths generated by the original system) not only at small time scales, but also at large time scales, under suitable assumptions on the dynamics. We discuss applications to homogenization of rough diffusions in low and high dimensions, as well as relatively simple systems with separations of time scales, and deterministic chaotic systems in high-dimensions, that are well-approximated by stochastic diffusion-like equations. In the second scenario we consider a system of interacting agents: given only observed trajectories of the system, we are interested in estimating the interaction laws between the agents. We consider both the mean-field limit (i.e. the number of agents going to infinity) and the case of a finite number of agents, with the number of observations going to infinity. We show that at least in particular cases the high-dimensionality of the state space of the system does not affect the learning rates. We exhibit efficient algorithms for estimating the interaction kernels, with statistical guarantees, and demonstrate them on various examples.

Deep Representations with Applications in Cryo-Electron Microscopy and Data Science
Roy R. Lederman, Yale University, USA

One premise of data science is that as datasets grow, they gain the potential to organize themselves in ways that reveal complex structures. The empirical success in recent years in using or recovering such structures may suggest that there are some fundamental properties of data, abstract models used to represent these data, and algorithms for data analysis, which we have yet to formulate. I will present examples where an interplay between representations, algorithms and massive datasets reveals hidden structures. The first example combines interpretable physical models with more abstract data organization to recover flexible molecular structures in Cryo-Electron Microscopy (cryo-EM). The second example uses more generic "deep networks" to discover hidden invariances, with applications to exploratory data analysis where we have more limited prior knowledge about the physical problems.

Progress on singular Brascamp-Lieb inequalities
Christophe Thiele, Hausdorff Center for Mathematics, Germany

Brascamp-Lieb inequalities are basic multi-linear inequalities generalizing, for example, Holder's inequality, and Young's convolutional inequality. Much progress in recent years has provided a satisfactory understanding of Brascamp-Lieb inequalities. Replacing one of the input functions by a Calderon-Zygmund kernel yields singular Brascamp-Lieb inequalities. They generalize, for example, Coifman-Meyer multilinear operators. Singular Brascamp-Lieb inequalities are much less understood than the non-singular case. We discuss a few examples and report on recent progress on understanding cases with enough symmetries.

From Wavelets to Diffusion geometry - reflections of some efforts handling large data sets
Jan-Olov Strömberg, KTH Royal Institute of Technology, Sweden

Already Fourier represented functions by their coefficients in his wellknown orthonormal system. When the concept of wavelets appeared, methods using represetation of functions by their wavelet coefficients gave rise to many applications. I have had the privilege to collaborate with professor R.R Coifman for many years and learnt a variety of wavelet methods and applications from him. One way to extend the methos is to adapt the orthonormal system to a function, or a set of functions by a selection procedure, choosing from a library of wavelet like function. The methods above are not feasible for very large data sets in high dimension. One way to study some large data set is Diffusion geometri. The Diffision map is generated by the data itself and can give information of strucure hidden in the data. I plan to illustrate Diffusion geometry geometry by some simpified examples.

Graphs, Eigenfunctions, and Metrics in Artificial Intelligence
Peter W. Jones, Yale University, USA

When searching for structures in large data sets, a well-known first step is to build a graph of the data set. The graph can then then be embedded in a low dimensional Euclidean space, and there are several algorithms for doing this. Our first topic is a review of why eigenfunction coordinate systems often provide “good dimensional reduction” in the sense that features in the data set are reproduced by the low dimensional embedding. One problem with this algorithm is that is not obvious which eigenfunctions to choose. We will presenta new version of eigenfunction coordinates, where one first projects onto RN using the (values of) the first N eigenfunctions. We introduce a “canonical choice” for producing a metric on the new data set lying in RN. Another topic to be discussed is the use of the Mahalanobis distance (i.e. metric) on Euclidean data sets. Ronald Coifman has used this in his works where one seeks “outliers” in the data. We will discuss a new version of this where the metric varies from scale to scale. The construction is a modification of work started by Xavier Tolsa. (Joint work with Raanan Schul and Gilad Lerman)