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Scientific Background on the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2018

ECONOMIC GROWTH, TECHNOLOGICAL CHANGE, AND CLIMATE CHANGE

The Committee for the Prize in Economic Sciences in Memory of Alfred Nobel

Economic Growth,
Technological Change, and Climate Change

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1 Introduction

This year's prize rewards the design of models and methods to address some of the most fundamental and pressing questions of our time, involving the long-run development of the global economy and the welfare of its citizens. Paul M. Romer has given us new tools for understanding how long-run technological change is determined in a market economy, while William D. Nordhaus has pioneered a framework for understanding how the economy and climate of our planet are mutually dependent on each other.

In his focus on the fundamental endogeneity of technological change, Romer has emphasized how the economy can expand the boundaries – and thus the possibilities – of its future activities. In his focus on the fundamental challenges of climate change, Nordhaus has stressed important negative side effects – and thus the restrictions – of the endeavors to bring about future prosperity. Both Romer and Nordhaus emphasize that the market economy, while a powerful engine of human development, has important imperfections and their contributions have thus offered insights into how government policy could potentially enhance our long-run welfare.

Expanding the domain of economics: knowledge and nature In central ways, the work by both Laureates draws on and overlaps with other sciences. Whereas advances of technology and engineering – broadly speaking, technical knowledge – had usually been taken as given by economists, Romer saw the frontiers of knowledge as also having central economic determinants. Similarly, Nordhaus recognized that the global climate – broadly speaking, nature – is not just an important determinant of human activity, but is simultaneously affecting society and affected by its economic activity. Thus, the two laureates have brought knowledge and nature into the realm of economic analysis and made them an integral part of the endeavour.

Long-run issues Romer's and Nordhaus's prize-winning contributions belong to the field of *long-run macroeconomics*. In textbooks, macroeconomic analysis is usually defined over different time horizons. Most well-known is the short-run perspective on the macroeconomy: the study of business cycles – the ups and downs of output over, say, a 10-year horizon. In the midst of such ups and downs, it is easy to forget the long-run perspective: the study of economic growth – the development of output, and more broadly human welfare, over decades or even centuries. Even small year-to-year differences in growth rates, which may seem tiny in a short-run perspective, cumulate. If such differences are systematic over decades, they build up to significant changes in living standards. Long-run macroeconomic performance is thus a dominant driver of the welfare enjoyed by current and future generations.

Market failures Romer's and Nordhaus's findings regarding the possibilities for, and restrictions on, future long-run welfare each put the spotlight on a specific market failure. Both laureates thus point to fundamental *externalities* that – absent well-designed government intervention – will lead to suboptimal outcomes. In Romer's work, these externalities are predominantly positive through knowledge spillovers. New ideas can be used by others to produce new goods and other ideas.[1] In Nordhaus's work, they are predominantly negative through greenhouse-gas emissions that adversely change the climate. In both cases, the externalities are not properly taken into account by the individual innovator or polluter, absent policy interventions such as subsidies/support for knowledge creation or taxes/quotas on emissions. Qualitatively, this conclusion goes back to Pigou (1920), but to devise the right dose of the right medicine requires models of the sort that the laureates pioneered.

Global issues In both cases, these externalities – and the resulting case for policy interventions – are *global* in nature and long-run in scope. Wherever its origin, an additional idea (blueprint) for a new technology can, in principle, be used anywhere else for the production of new goods and other ideas, contemporaneously or in the future. Similarly, an additional unit of carbon-dioxide emission, wherever its origin, quickly spreads in the entire atmosphere and roughly half of it will stay there hundreds of years and a substantial share much longer, contributing to global warming. In this sense, both prize-winning contributions deal with long-run, global, and sustainable growth.

A common stepping stone Moreover, the contributions by both laureates take a common starting point in the neoclassical growth theory for which Robert Solow was awarded the 1987 Economics Prize. Each of them extends this framework further in a significant and fruitful direction. Put succinctly, Romer provides the necessary add-ons – a set of knowledge-creation drivers – for understanding the determinants of long-run GDP growth, while Nordhaus incorporates the necessary add-ons – a set of natural-science mechanisms – for understanding how the global economy and the global climate interact.

[1] That the spillover is positive is not meant to say that all new ideas and products in reality are beneficial to mankind. The reader can probably come up with examples of welfare-reducing ideas.

Romer and Nordhaus thus highlight the strengths of Solow's original framework, namely its applicability to a host of important issues. But their research also rectifies two important shortcomings of his framework.

Endogenizing technological change In his approach to understanding economic growth over decades and centuries, Solow assumed an exogenous steady path for technology – the ultimate source of economic growth and well-being. In this sense, he did not address the very root of long-run growth. Romer, instead, focused precisely on the crux of how market economies might develop new technologies through profit-oriented research-and-development (R&D) efforts.[2] His solution laid the foundation of what is now ubiquitously referred to as *endogenous growth theory*. This theory argues that "ideas" are crucial for economic growth, and elaborates on the preconditions for the production of ideas.

New ideas, Romer argued, are very different than most economic goods by being *non-rival*: one person's use of an idea does not preclude others from using the same idea. But he also went on to emphasize another aspect of ideas: the extent to which they are *excludable*. Even if an idea can be used by two firms at the same time, it may be possible to exclude one of them from this use, either by regulation/patent law or by means of technical protection (e.g., via encryption). Excludability is critical for ideas to be produced in the marketplace, Romer reasoned, and not all ideas allow it. For instance, some forms of basic research do not fall in this category and may, hence, best be produced in universities.[3]

Next, Romer argued, the production of ideas typically entails increasing returns to scale, with large initial costs for the blueprint and low, arguably constant marginal costs for later replication. Romer thus emphasized that ideas and market power go hand in hand: market power is the typical way in which higher-than-marginal cost prices can be guaranteed, allowing firms to recoup the fixed costs of blueprints. In this sense, monopoly profits is the engine of market R&D. However, the fundamental non-rivalrousness of a productive idea can be regarded as a (potential) positive spillover – a positive *externality*. As the market solution involves both a degree of monopoly power and an externality, it typically generates an inefficient outcome. In summary, unregulated markets will produce technological change, but will not do so efficiently. This points to a potentially important role for economic policy, not just within each country but worldwide.

Endogenizing climate change Solow's original framework also did not consider any limits or obstacles to growth along a path of continuous economic development. Nordhaus has a long-standing interest in such growth obstacles at the global level, e.g., the finiteness of natural resources.[4] However, his deepest and broadest contribution concerned the obstacles

[2] Romer can perhaps be said to have developed and formalized the idea put forth by 1993 Economics Laureate Douglass North (1981) that market R&D has been crucial for the technological take-off of the developed economies into the modern growth era.

[3] Whether universities are financed publicly or privately is not central for this argument. Aghion, Dewatripont and Stein (2008) discuss the relative advantages and disadvantages of academic and private-sector research.

[4] See, for example, Nordhaus (1974).

due to climate change, which drew heavily on insights from different fields of natural science. In this realm, Nordhaus extended Solow's model with three important mechanisms: (i) how carbon concentration in the atmosphere depends on economic activity via carbon emissions, (ii) how global temperature depends on atmospheric carbon concentrations via increased radiation, and (iii) how economic activity and human welfare depend on global temperature via damages of many different sorts and strengths.

In this interdisciplinary fashion, Nordhaus developed *Integrated Assessment Models* (IAMs), the first generation of which is the Dynamic Integrated Climate Economy (DICE) model. IAMs allow us to assess different economic growth paths and their implications for the climate and, ultimately, the well-being of future generations. In these dynamic models, emissions reflect the burning of fossil fuels for economic use, and shape future well-being via the logical chain: carbon emissions ⇒ higher atmospheric carbon concentration ⇒ global warming ⇒ economic damages. In the same way as for R&D and knowledge creation, the market economy generates inefficient future outcomes at the global level. *The Stern Review* (2007) expresses this idea in a sharp way:

> "Climate change is a result of the greatest market failure the world has seen."

These market failures suggest that government interventions, via policies such as carbon taxes or emission quotas with a global reach, could be very valuable. The IAMs constructed by Nordhaus – and others who have followed in his footsteps – allow us to numerically compare different paths for future growth and well-being for different paths of policies.

The need for further research While Romer's and Nordhaus's research constitute critical steps forward, they do not provide final answers. But their methodological breakthroughs have paved the way for a great deal of further research (by themselves and by others) on global, long-run issues. Their analyses have laid bare a number of key areas where our knowledge is particularly weak. The frameworks they have built provide a structure to guide future research that may close these knowledge gaps. Follow-up research on technological change and the climate-economy nexus is very much an ongoing endeavor that has already led to important findings. But much more remains to be done.

The agenda on climate change and growth Nordhaus's methods show us the principles of how to analyze growth and climate change from a cost-benefit perspective. However, his analysis also shows the importance of measuring the damages of climate change and the uncertainty surrounding these damages. Research on these measurement tasks is still in its infancy. A first task, which is as daunting as it is necessary, is to "cover the map of climate damages" due to the vast heterogeneity and uncertainty about how – and through which channels – a changing climate affects different regions of the world.

A related task concerns "adaptation": how will human populations and their societies adapt to different climates, e.g., through migration? Technological change is another important adaptation channel. As Romer has taught us, such change reflects purposeful economic activity. Models built on his basic tenets can therefore help us analyze the incentives for developing technologies to facilitate adaptation and how policy might help redirect technological change.

Nordhaus's analysis also points to the importance of other concerns. Given the large uncertainties about future climates, thinking about appropriate policies involves – explicitly or implicitly – taking a stance on risk and uncertainty. Likewise, any policy considerations involve taking a stance on discounting. Since the effects of carbon emissions are much more long-lived than humans, it becomes critical to value the welfare of future generations. On both accounts, moral values may be necessary to complement scientific measurements. What models can do is to translate different value judgments into different paths for policy.

The agenda on technological change and growth Romer's early work had an enormous impact on research about economic growth, by pointing to shortcomings of the frameworks available in the late 1980s. Thus, his work set off a large number of theoretical and empirical studies aimed at understanding observed growth experiences.

While Romer's key breakthrough (Romer, 1990) envisioned innovation that expanded the variety of goods, other researchers (e.g., Aghion and Howitt, 1992, and Grossman and Helpman, 1991a) applied similar insights to the gradual improvements of a fixed set of goods. This alternative *creative-destruction* approach is very important in its own right, and emphasizes how an innovating firm can replace an existing firm by producing a given good at lower cost. Another important theory, building directly on Romer's ideas, concerns *directed technical change,* where resources spent on different kinds of R&D reflect market forces. One influential study (Acemoglu, 1998) shows how large cohorts of college-educated workers in the United States triggered research into technologies complementary with high-skill workers. This line of work helps us understand the rising wage inequality in some economies.

Differences in growth rates across countries and time periods was a central motivation behind Romer's key contributions. Because the central convergence prediction from Solow's basic framework seemed absent in the data, Romer's work marked the starting point of an increasingly vibrant literature that examined the data more carefully to contrast different theories of long-run growth. This empirical literature saw several waves based on different methods, including "growth regressions" focusing on convergence, structural assessments based on "development accounting," and approaches based on "natural experiments" to identify causal drivers of relative growth.

Romer's initial hunch was to see relative (long-run) growth rates of individual countries as endogenous to their own institutions and policy choices. Subsequent empirical research has stressed endogenous relative *levels* in the cross-section of national incomes. This empirical research is very much ongoing, and focuses on relative technological adaptation and innovation, human capital improvements, physical capital accumulation, and institutional conditions in general. Arguably, there is no commonly accepted "magic bullet". Just as short-run fluctuations can be spurred by different events at different points in time, long-run level or growth differences can have different explanations in different contexts. The international growth puzzle will perhaps never be fully solved, but it is much better understood today than it was in the early 1990s.

Organization of this overview Since both laureates start out from the neoclassical growth model, we begin (Section 2), with a brief reminder of its original components, along with the savings theory that dominates current macroeconomics. Against this common background, we first cover Romer's main contributions towards endogenizing the creation of ideas for new technology (Section 3), and then Nordhaus's main contributions towards combining growth and natural-science mechanisms into integrated assessment models (Section 4). Section 5 concludes.

2 Solow's Neoclassical Growth Model

The macroeconomic setting involves four key components: (i) a resource constraint, closely related to our system of national accounts whereby output (GDP) is allocated to its different uses, notably consumption and investment; (ii) a production function, describing how GDP is produced from its basic determinants, capital and labor; (iii) an equation describing the accumulation of capital; and (iv) a specification of how much of GDP is used toward investment and, hence capital accumulation. These four elements are presented first in the section. Romer and Nordhaus also include a model of saving behavior that goes beyond the one used by Solow; this model is presented next.

2.1 The Growth Model

The Solow model (Solow, 1956, and Swan, 1956) stays close to the national income and product accounts by first specifying a resource constraint. It assumes that the economy has only one good and tracks the production and use of this good over time. The model has been developed in a number of directions (allowing different goods, types of capital, and so on) and its main conclusions are, broadly speaking, robust to these extensions. Here, we focus on the basic version, partly to simplify the presentation, partly to follow Romer and Nordhaus who both employed that setting.

The resource constraint The resource constraint in year t reads

$$c_t + i_t = y_t,$$

where c is consumption, i investment, and y output. This constraint simply expresses how GDP is spent on these two components. Here, we will also think of it as an accounting equation for a single, economy-wide good, which can be used either for consumption or investment. The national accounts also include other components: government spending and net exports. Government spending can be thought of as subsumed in c and i. Net exports are relevant if one considers one of many economies in an international context. Solow's instead considered a "closed" economy, i.e., one that does not interact with the outside world. This view may seem wholly inappropriate when modeling individual countries today, given the existing amount of intertemporal and intratemporal trade. But it is a natural first step in Nordhaus's work, as the domain of his study is the world as a whole. We can also think of Romer's work as especially pertinent to a global analysis.

The production function Production of the single good is assumed to take place according to an *aggregate production function* F of capital and labor input:

$$y_t = F(k_t, l_t, t).$$

Here, k is capital and l labor input, and the third argument in the function is time, representing changes in production possibilities – especially improvements due to technological change – over time. The production function is strictly increasing in capital, $F_k > 0$ and labor, $F_l > 0$, and has decreasing marginal products of each factor: $F_{kk} < 0$ and $F_{ll} < 0$. Moreover, F has constant returns to scale in k and l – i.e., if k and l are multiplied by the same number λ, output rises by exactly λ. Solow, finally, assumed that production possibilities improve through *labor-augmenting* technical change: $F(k_t, l_t, t) = F(k_t, (1+\gamma)^t l_t)$, where $\gamma > 0$ is the exogenous rate of technical progress.

Capital accumulation and constant savings The capital-accumulation equation is straightforward:

$$k_{t+1} = (1-\delta)k_t + i_t,$$

where k is the capital stock and δ the annual rate of physical depreciation of this stock.

Finally, we need an assumption about how the investment, or saving, rate is determined. The Solow model assumes that $i_t = sy_t$, where s is the (exogenous and constant) rate of saving. Solow considered a constant rate of population growth. Under these assumptions, Solow showed that one obtains *balanced growth* asymptotically – i.e., the growth rates of c, y, and k converge to a common value.[5] Under a constant population this common growth rate is γ. In other words, if the stock of labor-augmenting technology grows exogenously at rate γ, so will the macroeconomic variables in the long run.[6]

2.2 Savings and Model Solution

The core model of saving in economics assumes that consumers save in a forward-looking, rational manner. Much of current macroeconomic analysis uses this approach, too, but there are a number of ways to summarize consumption behavior.[7] One concerns the extent of consumer heterogeneity in the population; another concerns the preferences over time and towards one's offspring. We will use the same assumption as in Romer's and Nordhaus's

[5]For this result, F_k has to be high enough for low values of k and low enough for high values of k.

[6]Given the long U.S. history of remarkable, stable and balanced growth at around 2% per year, this matched the U.S. data rather well. Reliable data for the world as a whole does not allow a long time series, but the available data is broadly in line with that of the U.S.

[7]For a background, see, e.g., Royal Swedish Academy of Sciences (2004).

models, which is also the most common one. This is to consider aggregate consumption as if it was made by a "representative consumer" who acts as a dynasty – i.e., she values the welfare of her offspring and does so in a manner consistent with how this offspring values her own welfare. Alternative assumptions can be entertained as well, without changing the analysis in a fundamental way. We now describe the details.

Optimal savings The literature following Solow's seminal work considered the optimal choice of saving from the perspective of maximizing consumer welfare. As mentioned above, we will consider a dynasty, meaning a family tree where c_t represents the total amount that the dynasty – with its different members – consumes at date t. The dynasty's utility function is given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t),$$

where u is an increasing and strictly concave power function.[8] The *discount factor* β is assumed to be less than one and represents the constant rate at which future utility flows – for oneself and one's offspring – are down-weighted on an annual basis. In the dynasty context, one can interpret $\beta < 1$ as impatience, in the sense that a given person puts higher weight on current than on future utility flows, but also that members of the current generation put lower utility weights on future generations (in its own dynasty) than on themselves.

It is possible to show that if one chooses the sequences of consumption, investment, capital, and output optimally subject to the resource constraint and the capital-accumulation equation, then the rates of growth of c, y, and k converge to γ, and the saving rate, $s_t \equiv 1 - c_t/y_t$, which is now endogenous and time-dependent, converges to a constant. In other words, Solow's assumption that the savings rate is constant rate follows from utility maximization. The optimization version of Solow's model is often referred to as the optimal-savings problem.[9]

Solving the model with optimal savings The more recent literature recognized that the optimal-growth outcomes – the paths for the macroeconomics variables – were derived from a model without frictions, such as externalities, or other reasons why the price mechanism might fail. Hence it became straightforward to present a market-based version of the growth model with endogenous saving, a dynamic competitive equilibrium, that delivered the exact same paths for macroeconomic variables as those chosen by a "benevolent social planner". In such a model, for example, a consumer would work for a firm, receive a wage income, and then optimally – from her dynasty's perspective – divide this income into consumption and saving on a market for borrowing and lending, where the interest rate is beyond the

[8]Such a utility function u – e.g., $\frac{c^{1-\sigma}-1}{1-\sigma}$ for $\sigma > 0$ (the case $\sigma = 1$ can be interpreted as $\log c$) – captures the idea that consumers strictly prefer more over less consumption. It also assumes that they enjoy each additional consumption unit less and less: marginal utility u_c is decreasing.

[9]The optimal-savings problem goes all the way back to Ramsey (1928), who was the first to study a dynamic optimization problem for saving, and the two papers first applying this principle for the neoclassical growth model: Cass (1965) and Koopmans (1965). Koopmans was awarded the Economics Prize in 1975.

consumer's control. Demands for capital and labor come from firms buying inputs in perfectly competitive input markets; they would also sell their output under perfect competition. Prices, i.e., the wage and the interest rate, can then be determined in each time period so that markets (for final output, labor, and capital, respectively) clear.[10]

Further, it is commonly assumed in macroeconomic analysis that the production function takes a specific form: $F(k_t, A_t l) = k_t^\alpha (A_t l)^{1-\alpha}$, where $\alpha \in (0,1)$ and A_t is shorthand for the current level of labor-augmenting productivity, i.e., $A_t = (1+\gamma)^t$ at time t. This so-called Cobb-Douglas production function is not only a mathematically convenient way of embedding the above-mentioned assumptions. It also delivers a property that is approximately satisfied in historical data for the United States (and many other countries), namely that labor's marginal product times its total amount (labor's total real income if the wage is equal to its marginal product as it is under perfect competition) is a constant fraction $1-\alpha$ of output regardless of the specific values of capital and labor.

Let us assume a Cobb-Douglas production function and a utility function

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma},$$

which features a constant elasticity of intertemporal substitution given by $\frac{1}{\sigma}$.[11] Then, we can summarize the predictions of the Solow model with endogenous saving as the solution to the following problem:

$$\max_{\{c_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma} \tag{1}$$

subject to

$$c_t + k_{t+1} - (1-\delta)k_t = k_t^\alpha \left((1+\gamma)^t l\right)^{1-\alpha} \quad \forall t = 0, 1, \ldots.$$

Romer and Nordhaus augmented this framework in fundamental ways. We now turn to their contributions.

3 Endogenous Technical Change

Solow's growth model was designed to capture three key aspects of long-run growth in the United States and elsewhere. Though systematic long-run data on macroeconomic aggregates were scarce at the time, some "stylized" facts were available. These facts were, in particular, (i) a rather stable output growth (y_{t+1}/y_t) , (ii) a stable capital-output ratio (k_t/y_t), and (iii) a stable ratio of consumption (or investment) to output (c_t/y_t).[12] Solow's theory had the *convergence* property, i.e., under the assumptions described above, no matter what the initial capital stock is, properties (i)–(iii) characterize the economy's long-run growth path.

[10] Labor supply is assumed to be exogenously fixed at l – i.e., a constant labor force with a constant utilization rate. It is straightforward to endogenize labor supply.

[11] The elasticity of intertemporal substitution is defined as $-\frac{u'(c)}{u''(c)c}$. This quantity must be constant for the model to be consistent with the observation that there is no long-run trend in the return to saving.

[12] These are part of Kaldor's "stylized growth facts" – see Kaldor (1957).

Thus, from the perspective of Solow's theory it was not a coincidence that the economy had these features. Moreover, the specific values for the growth rate and the ratios to which the economy converged were easy to derive as a function of the model parameters. The growth rate, in particular, was simply equal to the exogenous parameter γ.

However, the Solow model also predicted that *ceteris paribus* poorer countries should grow faster and catch up with richer ones quite quickly. This prediction of absolute convergence across countries reflects rapidly falling returns to capital, when parameter α is set low enough to be consistent with properties (i)–(iii) – see further discussion below. Of course, the model could accommodate persistent growth-rate differences, if γ – the rate of technological progress – differs across economies. But these differences would simply be assumed, not explained, as technological change arrives exogenously from a black box.

The empirical starting point Romer's work was motivated by the data on macroeconomic aggregates and a more comprehensive cross-country data set which had just become available (Summers and Heston, 1984). Romer noted and emphasized that this data showed very persistent differences between countries, not just in their output per capita but also in their growth rates. Moreover, there was no evidence that poorer countries grew faster than richer ones. These properties are clearly visible Figure 1, which is drawn from Romer (1987b), and shows data for 1960 income (output) levels (relative to the U.S.) and subsequent 1960-1981 average growth rates for 115 countries.[13] Thus the absolute-convergence prediction from the Solow model was violated in a broader cross-section of countries. Prolonged periods of persistently different growth rates in output imply massive changes in relative prosperity across the world economy – clearly a first-order question in economics and, more broadly, for the modern world. Romer set the goal to develop new theory that could address the prolonged periods of different growth across countries.

[13] The pattern in Figure 1 is present also if the time period is extended to today.

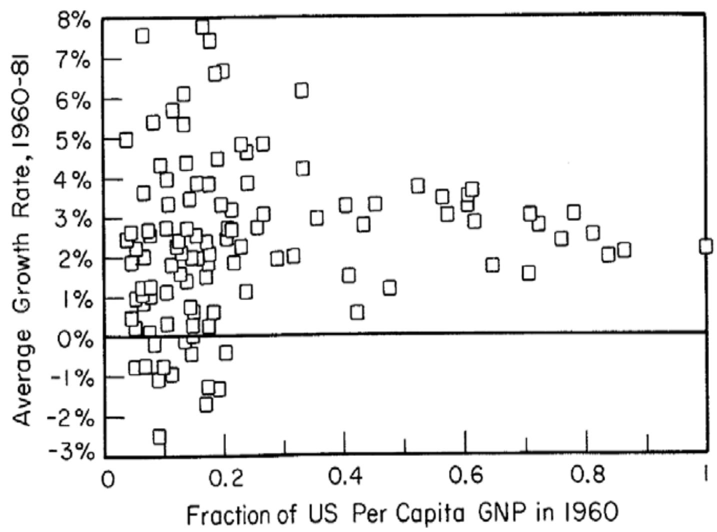

Figure 1: Growth in GDP per capita as a function of initial GDP per capita

Endogenous technology Romer's approach was to think about the determinants of γ in the framework just described. How might technology growth reflect conscious decisions to accumulate knowledge by agents acting on a market? Will it be constant over time or will it vary? How does it respond to incentives and economic policy, and should policy attempt to affect it? This line of attack on the problem posed formidable difficulties. One can formulate a social-planning problem where A, the level of technology, is chosen jointly with other inputs.[14] However, such a setting would be hard to study in a market context, at least under the typical assumption of perfect competition. The production function has increasing returns to scale if A is chosen as well. And an increasing-returns-to-scale production function is not compatible with perfect competition.

Romer's analysis of technology production, and conditions for it to occur in the marketplace, relied on thinking about knowledge creation at a more abstract level. He argued that "ideas", though produced with capital and labor inputs, are different than ordinary goods and services along two dimensions: the extent to which they are *rivalrous* – whether they can be used by more than one actor at once – and *excludable* – how easy it is to prevent others from using them. Romer emphasized that ideas are non-rivalrous and, to a varying degree, excludable. We will return to this point below, as it is of conceptual importance.

Romer also asserted that ideas go hand in hand with increasing returns to scale. They involve initially high costs, e.g., significant work for producing the blueprint (first copy) of a new product, but a more typical cost structure of (approximately) constant returns to scale for producing further copies. Hence, the overall production function is convex with

[14]Such an approach had been explored in the literature (see Shell, 1967).

falling marginal costs and one must therefore consider a departure from perfect competition. A key precondition for monopoly power is that the idea, or its use, must be excludable enough that a single firm can be the sole provider of the idea. Romer's most celebrated paper (Romer, 1990) worked these insights into a setting that contained the key elements – including monopolistic competition and increasing returns to scale – and built directly on Solow's workhorse model.

Sustained long-run growth Romer's 1990 formulation, and his papers more generally, emphasized the endogeneity of the long-run growth rate. To arrive at a technology-production model with this property, Romer not only incorporated the fundamental features of ideas discussed above. He also came up with a more technical, though highly influential, insight: the return to accumulated factors, such as capital, must remain strictly positive for the model to deliver sustained growth. For the equilibrium growth rate to be constant in the long run when growth comes from endogenous accumulation of a production factor, the accumulation technology has to be linear. This point is readily illustrated by the Solow model, where growth will peter out with decreasing returns to scale $\alpha < 1$, but will continue at a constant rate with $\alpha = 1$.

This technical point – which helped others produce endogenous-growth models of many varieties – is clearly worked out in Romer's first journal publication on growth (Romer, 1986). If the only change to the model is to make the production function linear in capital (say, by simply setting $\alpha = 1$ in the Solow model), however, it simultaneously makes non-accumulated factors, such as labor, less important. In his 1986 paper, Romer remedied this shortcoming by introducing a spillover effect of capital formation. As a result, growth came about as a by-product of regular capital accumulation, but with no explicit decisions to spend resources on R&D. We only briefly comment on this work at the end, in Section 3.3, and instead focus this section on the papers that took the more fundamental approach to model the production of new ideas.

Romer (1987a) first laid out a framework for new product development where growth was generated as a by-product of capital accumulation, but where an ever-expanding variety of intermediate goods prevented the returns on capital from falling to zero. In 1990, he showed how a close relative of the 1987 framework could be used to model R&D decisions in a decentralized market economy. This paper, Romer (1990), was a watershed. We introduce the discussion of Romer's watershed model with a brief description of the 1987 paper.

3.1 New Products and Capital Returns

Let us first try to understand why Solow needed to assume that growth ultimately came from technology growth, and the assumption that $\gamma > 0$. To see why capital accumulation could not lead to sustainable output growth by itself, let us consider the marginal product of a unit of capital: F_k. Even though the argument is more general, it is convenient to use the Cobb-Douglas production function for illustration. With this production function, we obtain $F_k(k_t, (1+\gamma)^t l) = \alpha k^{\alpha-1}\left((1+\gamma)^t l\right)^{1-\alpha}$. If $\gamma = 0$, F_k necessarily falls toward zero as the capital stock rises, making sustained growth impossible without technical change: even with a saving rate of unity, the long-run output of the economy could not exceed the finite value $\delta^{\frac{-\alpha}{1-\alpha}} l$ if $\gamma = 0$.[15] Intuitively, growth stops because of the decreasing marginal product of capital, which is a cornerstone of Solow's neoclassical theory. Further capital accumulation gives less and less and eventually capital depreciation exceeds its addition to production. By contrast, when the amount of labor in "efficiency units" rises, as it does when $\gamma > 0$, then the returns to capital accumulation are prevented from going to zero.

Love for variety Romer's idea was to think about how the returns to capital might be prevented from going to zero when capital grows without bound. In his 1987 paper, Romer thus presented the following alternative model to Solow's, where a "love for variety" and specialization allowed capital to earn a sustained positive return. Instead of having a homogeneous capital stock as an input, production comes about from (labor and) an *interval* of intermediate capital goods indexed by i: $x(i)$ is the amount of good i, and A is the endogenously determined length of this interval (which starts at 0). Total output is thus

$$y = \left(\int_0^A x(i)^\alpha di \right) l^{1-\alpha}, \tag{2}$$

where $\alpha \in (0, 1)$. In addition, assume that the length of variety interval (A) and the amount of each specialized capital good are determined by the existing amount of the standard (homogeneous) capital good at each point in time. The costs, in terms of homogeneous capital, of producing x units of a specialized capital good are convex and involve a fixed cost – they equal $(1 + x^2)/2$. Then, maximizing $\int_0^A x(i)^\alpha di$ over A and $x\,(i)$ given some available capital k implies that $A = (2 - \alpha)k$ and $x(i) = \sqrt{\frac{\alpha}{2-\alpha}} \equiv \bar{x}$ for all $i \in [0, A]$ and $x(i) = 0$ otherwise.

Quite intuitively, the presence of the fixed cost makes it optimal to choose a finite interval of length proportional to k. Due to the convex costs and the decreasing returns to each $x(i)$, it is optimal to assign an identical positive level of supply for each $x(i)$ in use. Inserting $A = (2 - \alpha)k$ and $x(i) = \bar{x}$ into (2) yields

$$y = (2 - \alpha)k\bar{x}^\alpha l^{1-\alpha},$$

where we recall that α, $\bar{x}$, and l are exogenous constants. That is, after maximizing over $x\,(i)$ and A, whatever the level of capital k available, output is linear in this level. This means that as capital is accumulated, its marginal product does not go to zero – it will in fact be constant at all times. As more capital is accumulated, the number of specialized capital varieties keeps going up, while each unit is used at the same level.

Allowing persistent growth The idea that variety expansion/specialization can allow capital to maintain its marginal product despite capital deepening allows growth to persist.

[15] To see this, note that with a saving rate of one, we obtain $k_{t+1} = (1-\delta)k_t + k_t^\alpha l^{1-\alpha}$. It is straightforward to plot this function and see that with $\alpha, \delta \in (0, 1)$ it converges monotonically to the stated value.

If we set investment to sy as in the original Solow model, the present setup delivers

$$k_{t+1} = (1-\delta)k_t + s(2-\alpha)\bar{x}^\alpha l^{1-\alpha} k_t.$$

Clearly, capital – and output – will grow at a positive rate at all times as long as $s(2-\alpha)\bar{x}^\alpha l^{1-\alpha} > \delta$; the economy's growth rate is the difference between these two expressions. The growth rate is thus *endogenous*: it depends nontrivially on the primitives of the model, including the savings rate.

This simple analysis shows how Romer managed to come up with an economic mechanism whereby capital accumulation, by its transformation into an ever-increasing variety of specialized capital goods, does not exhibit decreasing returns. At the same time, the analysis does not portray purposeful technology development. A slightly different version of the same model turned out to accommodate that interpretation, however.

3.2 The Production of Ideas

In his 1990 paper, Romer suggested that the following five properties would be desirable of a model of long-run economic growth.

1. The accumulation of ideas is the source of long-run economic growth.
2. Ideas are non-rival.
3. A larger stock of ideas makes it easier to find new ideas.
4. Ideas are created in a costly but purposeful activity.
5. Ideas can be owned and the owner can sell the rights to use the ideas at a market price.

As we have already seen, Romer emphasized the second and fifth properties: non-rivalrousness (which implies a form of positive externality) and partial excludability (which implies a monopoly distortion when implemented in a market economy). In Romer (1993), he described examples of products/services in these two dimensions with the diagram reproduced in Figure 2 below.

Clearly, not all ideas are excludable enough that a market solution would work – hence the need for a different form of ideas production (such as at universities). Romer did not fully explore the boundaries implied by this diagram – i.e., he did not formulate a theory (or test hypotheses empirically) regarding which ideas would be provided by markets and which would not. This remains an interesting research topic, in particular as one can imagine valuable ideas that are neither produced in the marketplace nor anywhere else.

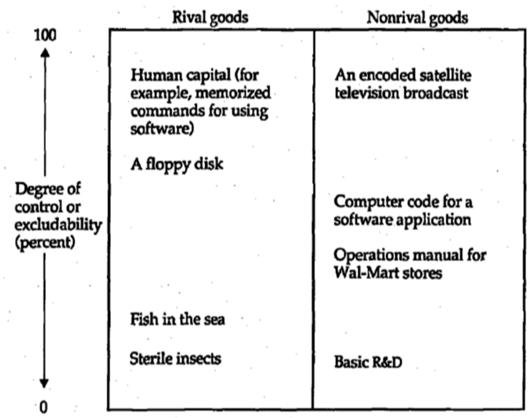

Figure 2: Goods and services: are they rivalrous and/or excludable?

Modeling ideas for new varieties Let us return to Romer's framework in Section 3.1, but incorporate the missing pieces. Let an idea in the model be a new variety i that cannot be used until it has been developed. It is developed in a costly process, which now uses labor (as opposed to capital in Section 3.1) as an input. In other words, labor can now be used in two ways. As before, it can be used to produce final output. But labor can also be used to produce new ideas, in which case we can think of labor inputs as research efforts. Let us assume that the cost of producing an idea is $1/(\xi A_t)$ units of labor. Denoting the number of researchers at time t by l_t^R, the number of new ideas – the variety expansion – is given by

$$A_{t+1} - A_t = \xi A_t l_t^R.$$

The fact that the productivity of researchers is proportional to the stock of existing ideas A_t is a simple way to incorporate Property 3 above in the model. This modeling feature also satisfies the linearity necessary for generating a constant long-run growth rate.

In the modified framework, a research idea i put to use is simply an amount produced of $x(i)$ – i.e., the specialized capital good. As before $x(i)$ is produced from a general capital good, but with a simpler – linear – production structure: to produce one unit of $x(i)$, η units of general capital are needed. With this assumption, we obtain the capital resource constraint

$$\int_0^{A_t} \eta x_t(i) di = k_t.$$

Given that each $x(i)$ has decreasing returns in final production, it is optimal to spread the general capital equally among the specialized goods: $x_t(i) = k_t/(\eta A_t)$ for all i.

In this setting, all ideas are equally good from a production perspective and their unit costs of production are also identical. In terms of Figure 2, Romer focused on a simple, symmetric setup that captured ideas that belong to the upper-right quadrant.

The planner's problem We can now state all the key equations determining quantities in this model. In particular, a benevolent social planner would solve the following problem:

$$\max_{\left\{c_t, k_{t+1}, A_{t+1}, l_t^R\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma},$$

subject to

$$c_t + k_{t+1} - (1-\delta)k_t = A_t \left(\frac{k_t}{\eta A_t}\right)^{\alpha} \left(l - l_t^R\right)^{1-\alpha}$$

and

$$A_{t+1} - A_t = \xi A_t l_t^R$$

for all $t = 0, 1, \ldots$.

The production function can be written as proportional to $k_t^{\alpha}\left(A_t l_t^F\right)^{1-\alpha}$, where $l^F \equiv l - l^R$ is the amount of labor used in final-good production.[16] Hence, the growth rate of A, namely ξl_t^R, is analogous to the exogenous rate γ in Solow's model, but here l_t^R is endogenous: it is the result of a choice that trades off the use of workers in final-output production against their use in research/ideas production.

Market R&D To see how markets may supply R&D, consider the producer of each specialized capital good i. Romer assumed that, in order for ideas to have value, they have to be granted patent rights. Thus, the production of good i requires a patent, which initially is bought from the inventor. In the simplest case, suppose patent rights are eternal. Then it is in the interest of the patent holder to be the sole producer, and it is in the interest of the inventor to sell the patent to only one producer. Hence Romer considered a monopoly producer of each good i.

However, since there are many competing capital goods and these are imperfect substitutes in production (perfect substitutes is the case when $\alpha = 1$), one can consider a framework with monopolistic competition, as in Dixit and Stiglitz (1977). To derive the demand function for each product i against which the monopolist will maximize profits, consider the final-good firms, which are assumed to operate in perfect competition. They maximize their profits, which can be expressed as follows.

$$\max_{(x_t(i))_i, l_t^F} \left(\int_0^{A_t} x_t^{\alpha}(i)di\right)(l_t^F)^{1-\alpha} - w_t l_t^F - \int_0^{A_t} q_t(i)x_t(i)di.$$

Here, w is the wage and $q(i)$ the price of specialized capital good i. Notice that the firm's problem is static and that w_t and the $q_t(i)$s are taken as given. The first-order conditions from this problem are

$$w_t = (1-\alpha)(l_t^F)^{-\alpha}\int_0^{A_t} x_t^{\alpha}(i)di \tag{3}$$

$$q_t(i) = \alpha(l_t^F)^{1-\alpha}x_t^{\alpha-1}(i). \tag{4}$$

[16] The constant of proportionality is $\eta^{-\alpha}$.

Equation (4) can be interpreted as the inverse demand function for good i. All other relevant prices are also taken as given, including the rental rate r_t paid for the capital that is rented from consumers. Then the owner of patent i obtains maximum profits $\pi_t(i) = \max_{k_t(i)} \{q_t(i)x_t(i) - r_t k_t(i)\}$ or, substituting from (4) and $x(i)\eta = k_t(i)$,

$$\pi_t(i) = \max_{k_t(i)} \left\{ \alpha (l_t^F)^{1-\alpha} \left(\frac{k_t(i)}{\eta} \right)^{\alpha} - r_t k_t(i) \right\}. \tag{5}$$

The first-order condition for this problem is

$$\alpha^2 (l_t^F)^{1-\alpha} \eta^{-\alpha} k_t(i)^{\alpha-1} = r_t. \tag{6}$$

Observe that $\pi(i) > 0$ is admissible: the firm owns a patent and obtains a rent from it, which makes the patent valuable.

The patent is produced by "R&D firms" in perfect competition. Let p_t^P denote the price of a patent at time t. Then ideas producers solve

$$\max_{A_{t+1}, l_t^R} p_t^P (A_{t+1} - A_t) - w_t l_t^R \tag{7}$$

$$\text{s.t. } A_{t+1} - A_t = l_t^R \xi A_t. \tag{8}$$

As Romer assumed free entry in the ideas industry, the equilibrium profits from engaging in research and development must be zero. Notice that this formulation has an implicit dynamic externality, sometimes labeled "standing on the shoulders of giants". The decision involving the change, $A_{t+1} - A_t$, raises the production of new ideas at all future periods, $t + j$, for $j \geq 1$, via the term $\xi A_{t+j'}$ for all $j' \in \{1, \ldots, j\}$ in the equation of motion for A. But this positive spillover effect is not benefitting the firm who chooses to change A. This dynamic spillover is the second reason why the planner's and the decentralized problems will have different solutions.[17]

The zero-profit condition in the ideas industry requires that the price p_t^P be determined from the first-order condition

$$p_t^P \xi A_t = w_t, \tag{9}$$

where w_t is the same as in the market for final goods: workers must be indifferent between which activity to join (research or final-goods production).[18]

Let p_t^C denote the relative price of consumption (final) goods at t (in terms of consumption goods at time 0). Then free entry implies that

$$p_t^P p_t^C = \sum_{s=t+1}^{\infty} \pi_s(i) \, p_s^C. \tag{10}$$

As a result of the equation just stated, no pure profits are generated in equilibrium. However, inventors of new patents appropriate the extraordinary rents that intermediate-goods producers will obtain from purchasing the rights on the invention.

[17] Recall that the patented goods are undersupplied due to the monopoly element.

[18] We consider one type of labor here for illustration purposes only. It would be more realistic to consider heterogeneity in worker skills, such that all workers do not have the option to become inventors.

Closing the market model Describing the consumer's problem in this economy is also straightforward. Consumers take the prices as given and are the ultimate owners of firms. They obtain profit incomes for the firms that have patents at time 0, but no net incomes for all firms created at time 0 and later. Consumers also accumulate capital and sell/rent it to the monopolistic firms. They also receive wage income from both final-goods firms and R&D firms. An equilibrium can then be fully defined to include all the conditions stated above.

It is instructive to combine the equilibrium conditions to a set of equations and compare them to the equations resulting from the solution of the planning problem. Such a comparison reveals that the market equilibrium has too little research and capital accumulation in equilibrium – compared to the efficient, planner-based allocation. Consequently, the equilibrium growth rate is too low, even though the existence of infinite-length patents provide incentives to do research in the market. Well-designed government policy, like subsidies to research, are necessary to rectify this market failure.

3.3 Romer's Capital Externality Model

As already mentioned at the outset of this section, Romer's 1986 first paper was the first in which the long-run growth rate is nontrivially determined and – at the same time – the equilibrium outcomes agree with a set of historical growth facts for the U.S. economy.

To see the contribution in Romer (1986), note that the simple so-called Ak version of Solow's model delivers an endogenous long-run growth rate. In this version, the production function is linear in capital, with no role for labor inputs. Linear production $y_t = Ak_t$ and capital accumulation $k_{t+1} = (1-\delta)k_t + sy_t$ gives a (short- and) long-run growth rate for both capital and output equal to $sA - \delta$, where A is a constant and s is the saving rate. Hence, if $sA > \delta$, this economy exhibits positive, and constant, long-run growth without any technological change. The reason is that the the marginal product of capital is not decreasing but constant at A. However, the model predictions fly in the face of other historical facts: not only does labor command a roughly constant share of firms' costs, but this share – around two-thirds – is considerable.

What Romer (1986) did was to formulate a simple model that had the Ak feature and hence an endogenous long-run growth rate, but was still consistent with the key historical growth facts. At the individual firm level, Romer assumes that $y_t = k_t^{\alpha}(A_t l)^{1-\alpha}$, where A_t is assumed to be equal to $\bar{k}_t$, capital used by one firm creates a positive spillover to all other firms. In equilibrium, we have $k_t = \bar{k}_t$ and $y_t = \bar{k}_t l^{1-\alpha.}$. As output is linear in capital, we have sustained growth. At the same time, the aggregate spillover comes for free and each individual firm only pays for the capital and labor they employ. As a result, the capital and labor shares of firm-level as well as aggregate costs accord with data – of $\alpha = 1/3$, these shares are one-third and two-thirds, respectively.

The key component in Romer (1986) was thus an Ak model with a positive labor share, derived in a decentralized equilibrium with externalities.

3.4 Subsequent Developments

Romer's early work had a deep and long-lasting impact on economic growth as an area within macroeconomics. As a testament to this fact, essentially all big-selling undergraduate textbooks used to be exclusively focused on business cycles up to the 1990s. Nowadays, they have much more contents on the topic of growth and some of these books even start out with long-run macroeconomics. In these textbook treatments, Romer's focus on idea production and the causes of technological change is now firmly established.

In the subsequent research literature, two distinct strands of work stand out. A very large set of research articles further theorize around the driving forces behind technological change and growth. This theoretical literature very clearly built directly upon Romer's work and developed it further in a number of directions. We discuss some of the most important developments in this subsection. Another, equally large literature is the empirical treatment of growth in a cross-country context. This empirical literature built only indirectly on Romer's work, although it was clearly inspired by it. We discuss this research more briefly in the next subsection.

Alternative drivers of endogenous growth Different theoretical follow-ups were pursued. One direction of research was inspired by Romer's discussions of decreasing returns to capital as an obstacle to long-run growth in the absence of technological change. For example, Rebelo (1991) presents a framework where capital goods – aggregate investment – is produced in a highly capital-intensive fashion. In particular, labor is not used at all in the capital-goods sector, and the production of new investment goods is hence linear in the capital stock employed there. By contrast, the consumption-goods sector has the standard form with a labor share of two thirds. Rebelo shows that such an economy will display long-run growth without technological change because the accumulable factor, capital, is produced without decreasing returns. His research can be viewed as a follow-up on Romer (1986).

A similar extension is to consider other accumulable factors in production. If the accumulable factors, jointly, can be reproduced linearly, the economy also displays perpetual growth without technological change. Such an approach was pursued independently and concurrently with Romer's early work by Robert E. Lucas, Romer's dissertation advisor and 1995 Economics Laureate. He developed a theory of human capital as the driver of growth, along with physical capital accumulation (Lucas, 1988). The continuous and endogenous building up of human capital – essentially augmenting the labor input in Solow's framework – prevents the returns from capital from falling, thus allowing continuous accumulation of physical capital as well. Lucas's work is not based on Romer's, but it shares the endogenous-growth feature.

Stokey and Rebelo's (1995) paper displays a very tractable version of the two-factor growth model, which is similar in spirit to both Rebelo (1991) and Lucas (1988). In other work along the same line, infrastructure appears as a separate input into production. This is treated as a government-provided good, largely because of its nature: a public good with the government as a natural producer. Here, there is perpetual growth at a constant rate if the joint return to infrastructure and regular capital is linear (the production function is homogeneous of degree one in the vector of these two stocks).[19]

Alternative R&D settings Another line of work provides alternatives to the specific R&D process that Romer (1990) laid out. The most influential contribution is the one in Aghion and Howitt (1992). Like Segerström, Anant and Dinopoulos (1990), they assume that new products replace old ones as perfect substitutes in use but at a lower production cost per unit. This mechanism is embedded in a growth model. The possibility to replace old goods implies that an innovator may "steal business" from a pre-existing firm and compete it out of the market. Also called *creative destruction*, this process is reminiscent of the one elaborated on at length by Joseph Schumpeter (1942), and it is clearly an important part of the driver of technological change. Aghion and Howitt (1992) shows that the existence of business-stealing has a very important implication: R&D and growth rates can be too high, since business stealing amounts to a negative spillover on existing firms.

A large literature on endogenous growth with creative destruction has followed Aghion and Howitt (1992) – for a general graduate-textbook treatment, see Aghion and Howitt (1998). Another key step in extending the theory was taken by Grossman and Helpman (1991a), who marry together insights from new growth theory with insights from new trade theory to analyze the relations between trade, innovation, and growth. Grossman and Helpman (1991b) provide a broad treatment of growth and innovation in a realistic setting where countries are part of a global economy.

In the wake of creative destruction, growth models have a rich set of predictions in the domains of industrial organization, exit and entry, competition and market structure, as well as for trade. These predictions have been pursued in a new wave of empirical research on innovation and growth, which often draws on microdata for individual firms.

A broad perspective on innovation recognizes that some innovations complement existing varieties (and not just substitute for them), whereas some substitute existing varieties by new, more efficient versions. A key factor is the degree of substitutability between old and new products. Moreover, pre-existing products may or may not be produced by the same firms that innovate. Whether the technology links are internalized thus depends on the precise market structure. A recent study by Garcia-Macia, Hsieh, and Klenow (2016), begins to try and assess quantitatively ow aggregate U.S. innovation can be accounted for by these different types of innovation.

Yet another path in the literature has been to consider decreasing returns to research, as an alternative to Romer's linear formulation. So-called semi-endogenous-growth models (see, in particular, Jones, 1995a, and Kortum, 1997), incorporate decreasing returns, but allow these to be counteracted by an increasing population. Hence, the long-run rate of technology growth becomes tied to the rate of population growth.

Directed technical change A separate theoretical extension considers how technological change is directed toward different uses. Acemoglu (1998, 2002), in particular, models how

[19] Barro (1990) has a similar model, where the flow of public expenditures acts as an input into production.

resources spent on different kinds of research are guided by market forces. These influential studies stress how large cohorts of college-graduated workers in the United States attracted research into technologies that are complementary with high-skill workers. This may have raised high-skill wages, despite the higher number of college graduates. What happens to the overall share of wages depends on the degree of substitutability between high- and low-skilled labor in production. Acemoglu argues that the substitutability is high enough to help us understand the rising wage inequality in most economies. This is an example of research on endogenous technology that builds directly on Romer's ideas. Some of the papers on directed technical change use the expanding-variety model of Romer (1990), while others employ the creative-destruction model of Aghion and Howitt (1992).

In more recent work, Acemoglu et al. (2012) apply the idea of directed technical change to an important topic in climate change, namely how much of R&D is devoted to technology research aimed at improving "green" (as opposed to "dirty") technologies. Here, Romer's techniques and insights are also used to conclude that subsidies to the development of green technology may help mitigate climate change by reducing the reliance on fossil fuel. Moreover, even a temporary policy could have a very powerful role, via the kind of permanent effects inherent in Romer's setting. That regulation may be required to direct market-based R&D towards developing ideas that are beneficial for welfare is a general conclusion. The notion of directed, endogenous technological change has been applied in other contexts as well, such as in trade theory.

3.5 Quantitative Evaluations

As argued at the outset of this section, Romer was motivated by the challenge to explain the available cross-country and time-series data on output growth, as illustrated in Figure 1. Displaying these data and pointing to the obvious – that the world economies seemed far from converging to a common level of output per capita – and showing how basic growth theory could be amended to account for the empirical patterns was a powerful ignition for empirical research. These kinds of data and the theorizing Romer put forth had not, for a long time, been central in economic research or in the teaching of macroeconomics. The situation today is very different, as a result of decades of empirical work on economic growth. What has the empirical literature found?

Growth redux As perhaps could have been expected, the empirical literature has not offered conclusive evidence on "the top drivers" of growth among countries. It has, however, generated many insights and reached considerable maturity. In the very brief discussion that follows, we emphasize our understanding of the current consensus on some first-order issues.

When it comes to *relative* growth performances, the consensus appears to be somewhere in between Solow's convergence-based theory and endogenous-growth theory. *Conditional* convergence appears to be a fact – i.e., countries with similar traits and policies tend to converge to a similar level of GDP per capita. Robert Barro is a key contributor towards establishing this consensus – see Barro (2015) for a recent summary.

However, convergence is much slower than implied by a straightforward calibration of Solow's model, where α is usually argued to be around 1/3.[20] In other words, consistently with Figure 1, a country's relative position can be drastically and persistently influenced by policy or other factors that make its growth rate depart significantly from the world average for quite some time. The consensus view also holds that sooner or later a country's growth rate will slow down to the world average: it is not possible to grow at a higher rate than the rest of the world for a very long time. Thus, going back to the Solow model and its parameters, the value of α appears much higher than previously believed, but less than 1. Furthermore, countries can influence their relative values of A and thus their position relative to the world frontier. Early research papers to emphasize elements of such a model economy were Mankiw, Romer, and Weil (1992) and Parente and Prescott (1994). Jones's influential undergraduate textbook in economic growth (Jones, 1998) is another example. The notion here is that the growth rate of the average A in the world, or of the As of the leading countries, is an endogenous function of world-wide investments in technology and knowledge creation.[21]

Put differently, the consensus view is that GDP per capita across countries has a rather stable overall distribution in *relative terms*, where (i) mean GDP per capita keeps growing at a stable rate, but (ii) the relative positions in the distribution are substantially reshuffled over time. In other words, we keep observing *growth miracles* along with *growth disasters*: long-lasting and large changes in relative positions, upwards as well as downwards. There is less consensus on what drives these miracles or disasters. Institutional factors, human-capital accumulation, and openness to trade, are often mentioned as prime candidates, although case-based analyses suggest that the relative importance of these factors differ widely across miracles and disasters.

Empirical tests of growth theory Empirical research on growth from a more global perspective has also been conducted, but is rare. Kremer (1993) examines a key implication of Romer's theory, namely increasing returns: societies with more people should produce higher growth rates. The hypothesis is hard to test since countries, for a long time, have been connected through trade and ideas exchange, so the unit of analysis can hardly be that of a country. Kremer therefore goes very far back in time and looking at isolated societies he finds support for Romer's theory. Jones (1997) works out the implications of the hypothesis that population size is key for long-run growth rates. More generally, a series of papers by Jones (1995a,b, 1999) evaluates the performance of endogenous-growth theories from an empirical perspective. More recent work by Bloom, Jones, van Reenen, and Webb (2017) documents, in particular, a significant extent of decreasing productivity of research, viewed from the perspective of the world-technology frontier.

An empirical challenge in assessing theories of technological change is the difficulty to

[20]Under perfect competition, this value is equal to capital's share of income.

[21]An early theoretical paper emphasizing the world determination of both these investment decisions and international spillovers of knowledge is Rivera-Batíz and Romer (1991), which studies a two-country model and shows the potential importance of trade for the world growth rate.

measure the number, and economic values, of innovations. Recently, microeconomic data sets for patents and patent holders have become available and there is now a vibrant literature testing different versions of endogenous-growth theory. Similarly, increasing access to census and register data for individuals makes it possible to identify "innovators" and "entrepreneurs", making it possible to test the specific microfoundations for different modules of R&D models. This important endeavor is a very active research area today. It has also become an important input into the discussion of the determinants of inequality – since much of the new riches are associated with returns to innovation and the associated entrepreneurship, also touching on the role of policy both from an innovation and inequality perspective.[22]

4 Integrated Assessment Models

Nordhaus laid the foundations for extending the Solow model to capture the long-run interactions between society and climate. His interest in these interactions goes back to the 1970s. At that time, natural scientists were paying increasing attention to the practical importance of a theoretical possibility: that the burning of fossil fuels to produce energy for production or consumption may significantly warm the world.[23] Moreover, they warned that a warmer climate could be detrimental in a variety of ways. Nordhaus closely followed these discussions and took on a task that was both daunting and pioneering, namely to model the interactions between economic growth and climate change.

General approach His over-arching idea was to consider how output and – more generally – human welfare would be constrained by changes in the climate due to the use of fossil fuels. Nordhaus argued that in order to analyze how the economy influences the climate, how the climate influences the economy, and how different policies influence the outcomes of interest, one must incorporate knowledge from the natural sciences into a suitable model of long-run growth.

To satisfy these requirements, a climate-economy model must be dynamic and include three interacting sub-models:

1. a carbon-circulation model that maps emissions of fossil carbon to a path for atmospheric carbon-dioxide (CO_2) concentration

2. a climate model that describes the evolution of the climate over time depending on the path of CO_2 concentration

[22]See, e.g., Akcigit, Celik, and Greenwood (2016), Acemoglu, Robinson, and Verdier (2017), or Aghion, Akcigit, Bergeaud, Blundell, and Hemous (2018).

[23]1903 Chemistry Laureate Svante Arrhenius had provided the first analysis of whether fluctuations in atmospheric CO_2 concentration are important enough to explain fluctuations in observed temperatures (Arrhenius, 1896). See further discussion below.

3. an economic model that describes how the economy and the society is affected by climate change over time, and – closing the loop – how the path of economic activity leads to emissions of fossil carbon.

Nordhaus showed how these very different sub-models could be integrated into one framework. We nowadays ubiquitously refer to such frameworks as *integrated assessment models* (IAMs). An IAM can make consistent projections. For example, it will simulate the future climate based on fossil-fuel emission paths produced from a global economic model that takes these same climate simulations as inputs. Consistency of the simulations is obviously not a guarantee for accurate forecasts. But it is nevertheless a desirable feature, especially if one wants to examine the effects of policy, since the policy effects involve a feedback from the economy, via the climate, back to the economy.

As noted in Sections 1 and 2, Nordhaus builds on the neoclassical growth framework – in a version parameterized to match historical macroeconomic data – with endogenous saving and explicit welfare functions. Given these welfare functions, the models can answer normative questions, e.g., about the desirable time path for a global carbon tax. Obviously, any normative conclusion reflects normative assumptions, such as welfare weights attached to people at different points in time and space. Given a set of welfare weights, the model can readily be used to identify "optimal" policy. When we speak about optimal policy below, we thus refer to using the model in that way, namely to quantify how (different) normative assumptions shape variables like carbon taxes, temperature limits, and emission paths.

Why such a simplified model? Nordhaus's approach was to condense and simplify state-of-the-art knowledge about global carbon circulation and the climate system into a set of (close-to) linear equations that was tractable enough to handle in an economic model. To understand the need for drastic simplifications of the natural-science elements of the IAM, note that the economic model assumes that agents are forward-looking. Indeed, people's concern with climate change is in itself evidence of the forward-looking capacity of humans. Like Romer, Nordhaus assumes rationality as a benchmark. Under rational forward-looking behavior, the optimality conditions that pin down the laws of motion for endogenous variables (including fuel prices and interest rates) imply that current variables like consumption depend on the entire path of future endogenous and exogenous variables.[24]

A key step in economic model building is therefore to *solve* the model. To do so, one needs to find a mapping from the "state", i.e., the predetermined variables (e.g., the capital stock and the level of technology at the beginning of a certain time period), to the endogenous variables (e.g., consumption) that satisfies the forward-looking conditions. This step is absent in models of climate and carbon circulation, as the differential equations that determine the model dynamics are recursive: they have no forward-looking components. That is to say, particles in natural-science models – unlike humans in economic models – do not choose

[24]For the arguments here to be valid, full rationality in forward-looking behavior is not essential. As long as agents are to some degree forward-looking, solving a dynamic economic model involves a fixed-point problem, the complexity of which rises very quickly with the number of state variables.

their paths based on expectations about future events (including how other particles will act, today as well as in the future).

This fundamental difference makes the standard large-scale models of climate and carbon circulation incompatible with economic models. Just joining a set of standard (sub)models, would yield a model much too complicated to solve, given the large state space of conventional natural-science models. The incompatibility is reinforced when models are used to find optimal policy, since the set of possible policies to consider and compare is very hard to reduce to a manageable size. For this reason, Nordhaus's demonstration that a compact and easy-to-analyze climate and carbon circulation system could be made compatible with a forward-looking economic model is a fundamental contribution. Obviously, the simplifications on the natural-science side have some costs. As nature is complex and non-linear, one must take care to avoid simplifications that lead to unwarranted conclusions. This is something Nordhaus has kept emphasizing ever since he started his research in the area.[25]

In what follows, we describe the key IAM models Nordhaus has built, their uses, and their further developments. However, we begin in Subsection 4.1 by briefly describing a precursor to his main achievement described in Subsection 4.2.

4.1 The 1975/1977 Provisional Model

We now summarize the model in Nordhaus (1975, 1977). This is not a full-fledged interacting IAM, as it lacks a climate model and an explicit formulation of economic damages from climate change. However, it is an important precursor of Nordhaus's later work. Its aim was to specify how the atmospheric CO_2-concentration – and thus climate change – could be kept at a tolerable level, at the lowest possible cost. Such an analysis remains valuable today when political goals have been set for climate change in the 2015 Paris Agreement under the United Nations Framework Convention on Climate Change to keep the increase in the global mean temperature below 2 degrees Celsius (2°C).

Carbon circulation A tractable description of how a path of CO_2 emissions translates into a path of atmospheric CO_2 concentrations is a necessary ingredient in an integrated climate-economy model. Modeling the relation between emissions and concentrations in turn requires an understanding of many complicated physical and biological processes, such as the photosynthesis, the gas exchange between atmosphere and ocean, and the mixing of different ocean layers. Nordhaus (1975) builds on Machta (1972), a paper presented at the 20th Nobel symposium "The Changing Chemistry of the Oceans". He thus constructs a model with seven different reservoirs of carbon, namely: (i) the troposphere ($<\simeq$ 10 kilometers), (ii) the stratosphere, (iii) the upper layers of the ocean (0–60 meters), (iv) the deep ocean ($>$ 60 meters), (v) the short-term biosphere, (vi) the long-term biosphere, and (vii) the marine biosphere. Based on findings from the natural sciences, Nordhaus argues that

[25] Of course, Romer's models of growth share these features: they are, in general, demanding to solve as they are dynamic and forward-looking. We emphasize the difficulties here, as they contrast with natural-science models and place restrictions on the climate and carbon-cycle modules of the IAMs.

the gross flows between these reservoirs can be approximated as proportional to their source reservoirs. For example, in Nordhaus's calibration, 11% of the carbon in the troposphere flows each year into the upper ocean and 9% of the carbon in the upper ocean flows in the other direction.

Given these assumptions, the carbon circulation can be modeled as a linear first-order system with a time step of one year

$$M_{t+1} = D \cdot M_t + E_t, \tag{11}$$

where M_t is a seven-element vector encompassing the sizes of the seven carbon reservoirs. D is a 7×7 matrix of flow coefficients where, e.g., the first element in the third row gives the yearly 9% flow from the upper ocean to the troposphere. The diagonal of D tells us how much carbon in each reservoir stays in that reservoir. The elements in each column sum to unity: no carbon is lost in the system. Finally, E_t represents emissions. Since all emissions go to the troposphere, E_t is a seven-element vector where only the first element is non-zero.

Using this model, one can describe the evolution of atmospheric CO_2 concentration (as well as the amount of carbon in the other reservoirs) for any emission scenario.

The economy The economic part of an integrated model should, at a minimum, predict a path for emissions and describe how different policies influence these emissions. To conduct a normative analysis – to rank different policies according to their desirability – the model must include some welfare measure. Nordhaus (1975) is a quite detailed framework for global energy demand. In contrast to his later work, it is formulated as a partial equilibrium model that takes the path of global GDP as given and uses this path as an input into the demand for energy. Energy is demanded for four different purposes: electricity, industry, residential, and transportation in two regions (the U.S. and the rest of the world). It is supplied from 6 different natural resources: petroleum, natural gas, coal, shales, and two types of uranium (U_{235} and U_{238}). The cost of extraction, conversion, and transportation, as well as geological availability, are taken into account.

Nordhaus later made significant efforts at estimating the damages (and gains) to society of climate change. At the time, however, aggregate summaries of such effects did not exist. Nordhaus (1975) therefore argued that a reasonable first step is to analyze how a constraint for atmospheric CO_2 concentration can be achieved at minimum cost. He careful notes that this exercise is not meant to describe how much climate change should be allowed. But the paper discusses likely consequences of different paths of CO_2 concentrations, including the effects on temperatures and sea levels, by using simplified climate models integrated with the rest of the model. In the absence of economic cost estimates, the model is used to calculate the economic costs of satisfying different scenarios for CO_2 concentrations.

4.2 The First Complete Model

Nordhaus's first fundamental, quantitative contribution was the construction of the DICE model (Dynamic Integrated model of Climate and the Economy). Published in Nordhaus

(1994a), this model lays the foundations for the IAMs still used today by, e.g., the Intergovernmental Panel on Climate Change (IPCC).[26] The first vintage of DICE used the latest knowledge from the natural sciences to construct a dynamic carbon-circulation system as well as a dynamic relation between changes in the global-energy balance and the global-mean temperature. These relations were on a form simple enough to be combined with a Solow model of economic growth, where the production of output uses fossil fuel, in addition to capital and labor, as in Dasgupta and Heal (1974).

Two years later, Nordhaus presented a modified model with a number of regions (Nordhaus and Yang, 1996), labeled RICE (Regional dynamic Integrated model of Climate and the Economy). Both DICE and RICE were adapted to the numerical program packages GAMS and EXCEL, so as to make the models transparent and easy to work with also for other researchers. Ever since their original versions, both RICE and DICE have been continuously developed and refined, by both Nordhaus and other scientists. They still remain the workhorse models for climate economics all over the world.

Next, we discuss the different components of DICE and RICE in more detail, beginning with the natural-science elements and going on to the economic elements.[27] First, we discuss how Nordhaus incorporates a climate model into the analysis – this part takes a path of atmospheric carbon concentration as input and generates a path of climate (global mean temperatures) as output. Second, we show the carbon-circulation model – a simplified version of the one described in the previous subsection. Third, we describe the explicit consideration of economic "damages", an addition to the model which is necessary to allow for explicit cost-benefit analysis. Fourth, we discuss the remaining features of the economic model, whose core (the Solow model) we have already described. Finally, we dwell on how to calibrate the model's parameters and solve the model, before putting it to use.

Climate For over 100 years, it has been known that CO_2 is a greenhouse gas that changes the energy budget between incoming sunlight and outgoing long-wave heat radiation. In fact, 1903 Chemistry Laureate Svante Arrhenius described the direct effect of CO_2 concentrations on the energy budget by the famous and still heavily used formula:

$$F_t = \frac{\eta}{\ln 2} \ln \frac{M_t}{M_0}. \tag{12}$$

Arrhenius's formula says that the change in the energy budget F, measured in power per area, is proportional to the natural logarithm of the ratio between actual and baseline atmospheric CO_2 concentration, denoted by M_t and M_0. Parameter η measures how the energy budget changes with a doubled CO_2 concentration. With this simplified representation of the greenhouse effect, Nordhaus formulated a system of difference equations for global mean (surface) temperature T_t and ocean temperature T_t^O, both expressed as deviations from their pre-industrial levels. These equations should be thought of as linear approximations around

[26] This organization was awarded the 2007 Nobel Peace Prize.

[27] The description below is based on DICE and RICE in Nordhaus and Boyer (2000). The minor differences from the original DICE and RICE models are pointed out below.

the pre-industrial steady state of a non-linear system based on the law of nature that energy does not disappear. We can label the system a *global energy budget* and the DICE energy budget is

$$\begin{aligned} T_t - T_{t-1} &= \sigma_1 \left((F_t + O_t - \kappa T_t) - \sigma_2 \left(T_{t-1} - T^O_{t-1} \right) \right) \\ T^O_t - T^O_{t-1} &= \sigma_3 \left(T_{t-1} - T^O_{t-1} \right), \end{aligned} \tag{13}$$

with a decadal time step.

The term $(F_t + O_t - \kappa T_t) - \sigma_2(T_{t-1} - T^O_{t-1})$ describes the energy budget of the atmosphere and the upper layer of the oceans. Here, F_t measures the additions of energy flows due to the CO_2 greenhouse effect, O_t includes other man-made additions, including methane and aerosol emissions, while κT_t quantifies the fact that a warmer body radiates more energy. In this case, the so called Planck feedback, implies that a warmer earth, all else equal, radiates more energy into space in the form of infrared light. The term $\sigma_2(T_{t-1} - T^O_{t-1})$ reflects energy flows from the atmosphere to the deep oceans, which is a function of the temperature difference and enters with a negative sign in the energy budget of the atmosphere. If the overall energy budget of the atmosphere and upper ocean is in surplus, the atmospheric temperature will rise: $T_t - T_{t-1} > 0$. For a given surplus, the speed of the temperature increase is determined by the heat capacity of the atmosphere and the upper ocean, parameterized by σ_1.[28]

The energy budget of the deep ocean is simpler, and only contains the energy flow from the atmosphere and upper ocean layer. If it is in surplus, i.e., net energy flows are downwards, $T_{t-1} - T^O_{t-1} > 0$, the oceans become warmer at a speed determined by σ_3.

It is straightforward to see that, if CO_2 concentration stabilizes at twice the pre-industrial level ($\frac{M_t}{M_0} = 2$), the addition to the energy budget is η. Disregarding other exogenous additions to the energy budget ($O_t = 0$), a new steady state will then eventually materialize in which the two temperatures are constant. For this to be possible, the atmospheric temperature must increase so as to balance the greenhouse effect:

$$T = \frac{\eta}{\kappa}.$$

The ratio $\frac{\eta}{\kappa}$ is often labeled the *equilibrium climate sensitivity.* Equilibrium here refers to to the long-run steady state response. The response over short horizons is called the *transient climate sensitivity,* specified for a particular horizon. Because many, more or less well-understood, feed-back mechanisms operate in reality, there is substantial uncertainty about this number. For example, the fifth IPCC report asserts that the equilibrium climate sensitivity is "likely in the range 1.5 to 4.5°C". In 1999, Nordhaus chose a value of 2.9 for $\frac{\eta}{\kappa}$ in DICE and RICE.

Carbon circulation The carbon-circulation model in DICE and RICE describes the dynamic evolution of the emitted CO_2 – thus it is closely related to the system in Nordhaus

[28]The heat capacity of the atmosphere alone is very low relative to that of the oceans, implying a rapid equalization of its own energy budget, had it been specified explicitly.

(1975) described in Subsection 4.1. It has only three reservoirs: the atmosphere (M_t), the biosphere and upper layers of the ocean (M_t^U), and the deep oceans (M_t^L). Here, the variables M_t, M_t^U, and M_t^L measure the mass of carbon in the respective reservoirs.[29]

Simplifying (11) into these three components – and exploiting the properties that carbon cannot vanish and that the inflows from one reservoir must be identical to the outflows from another – we can rewrite the carbon-circulation system as:

$$\begin{aligned} M_t - M_{t-1} &= -\phi_{12} M_{t-1} + \phi_{21} M_{t-1}^U + E_{t-1}, \\ M_t^U - M_{t-1}^U &= \phi_{12} M_{t-1} - (\phi_{21} + \phi_{23}) M_{t-1}^U + \phi_{32} M_{t-1}^L, \\ M_t^L - M_{t-1}^L &= \phi_{23} M_{t-1}^U - \phi_{32} M_{t-1}^L. \end{aligned} \tag{14}$$

Here, $\phi_{12} M_{t-1} - \phi_{21} M_{t-1}^U$ represents the net carbon flow from the atmosphere to the upper reservoir M_t^U, which is subtracted in the first equation and added in the second. Analogously, $\phi_{23} M_{t-1}^U - \phi_{32} M_{t-1}^L$, is the net the flow to the deep ocean from the biosphere and upper layers of the ocean, being subtracted in the second equation and added in the third.[30]

Having specified the dynamic model of carbon circulation, Nordhaus calibrated its parameters to make it behave in line with state-of-the-art carbon circulation models.[31]

Damages A key innovation in DICE and RICE over Nordhaus (1975) was the addition of a damage function mapping global-mean temperatures into economic damages. Nordhaus (1994a) pioneered the "bottom-up" approach to damage aggregation. His idea was to compile a large number of microeconomic studies on various consequences of climate change – e.g., the damages to agriculture, coastal regions, amenity values, biodiversity, and human health. Of particular interest is the "Ricardian" approach used by Mendelsohn, Nordhaus, and Shaw (1994), who use the relation between temperature and market prices of farm land across 3000 U.S. counties to infer the consequences of climate change, an approach that controls for many aspects of underlying institutions.

Nordhaus realized that such a bottom-up approach to measuring damages abstracted from some damages, in particular extreme outcomes with low probability. In the words of Nordhaus (1994b), "we have only 'best guess' scenarios for climatic change and the social reactions it might cause". He argued that very misleading policy implications could arise from a model that disregarded such best guesses of damages associated with climate change, even though little hard evidence for them could be presented. As a partial remedy, he conducted a survey among a carefully selected panel of scientists with particular expertise in climate change and its consequences.[32] This survey solicited assessments of potentially damaging consequences of climate change and their associated probabilities. The survey

[29] Since the ratio of current atmospheric concentration to the preindustrial level is equal to ratio measured in carbon mass, the Arrhenius law in (12) is the same for both variable definitions.

[30] Some flows, e.g., respiration and photosynthesis can be defined as gross flows, while others like the gas exchange between the atmosphere and ocean surface cannot.

[31] In particular, he used the BERN carbon circulation model also used by IPCC (1996).

[32] The names of the panel members are given in Nordhaus (1994b).

results were condensed into a "catastrophic-impact" component, including a risk premium (depending on what individuals are willing to pay over the expected value to reduce a risk).

Finally, Nordhaus specified a function

$$\Omega\left(T_t\right) \equiv \frac{1}{1+\theta_1 T_t+\theta_2 T_t^2} \tag{15}$$

to represent the share of GDP left after climate damages (regional for RICE and global for DICE). He chose parameters θ_1 and θ_2 to make this overall damage function approximate the sum of the underlying mechanism-specific damage functions, including the catastrophic impact component. The damage function describes how much society loses as a result of global warming, with less resources for consumption and investment.[33]

This damage function is part of the economic part of the model, which we now discuss in somewhat more detail.

The economic model As already mentioned, the economic models in DICE and RICE are based on the Solow model with optimal savings. As such, they include agents with an explicit utility function. In RICE, the world consists of eight regions: U.S., OECD-Europe, Other high income, Russia and Eastern Europe, Middle income, Lower middle income, China, and Low income. Consumers maximize their utility by choosing how much to save and consume taking prices as given. Specifically, in region j, consumer welfare is

$$\sum_{t=0}^{\infty} \beta^t L_{j,t} \frac{c_{j,t}^{1-\sigma}-1}{1-\sigma}, \tag{16}$$

where $L_{j,t}$ is the region's population in period t and $c_{j,t}$ is its per-capita consumption analogously to (1). $L_{j,t}$ increases initially in line with observed population growth, but population growth is assumed to fall over time, eventually leading to a stable global population. Often, the calibration of σ is unity implying a logarithmic utility function. As in the dynasty model discussed in Section 2, the infinite sum in the welfare expression reflects the assumption that individuals are altruistic across generations, effectively creating infinitely lived dynasties.

Firms maximize the discounted sum of profits also taking prices as given. They produce the final good and hire labor, capital and energy on competitive markets. As in the Solow model, they are assumed to use a Cobb-Douglas production function, where output of the final good in region j and period t is

$$\Omega_j\left(T_t\right) A_{j,t} k_{j,t}^{\alpha} l_{j,t}^{1-\alpha-\gamma_j} es_{j,t}^{\gamma_j}. \tag{17}$$

The inclusion of a finite resource – fossil-based energy services, es – extends the basic Solow model as in Dasgupta and Heal (1974). Total-factor productivity $\Omega_j\left(T_t\right) A_{j,t}$ has two terms. One is $\Omega_j\left(T_t\right)$, the net-of-damage function described in the previous section. This

[33] Some of the damages incorporated in Ω are not literally output losses, but loss of life, loss of amenity values, etc. Thus, Nordhaus's approach is to translate them into equivalent output losses.

gives rise to a negative externality, as no individual economic actor factors in how her individual decisions affect global temperatures. The second term is $A_{j,t}$, a technology parameter, which increases exogenously over time just as in the original Solow model (Nordhaus did not incorporate Romer's endogenous-technology mechanism). The amount of energy services per unit of carbon emissions are: $es_{j,t} = \xi_{j,t} e_{j,t}$, where $e_{j,t}$ is fossil energy use and $\xi_{j,t}$ is the inverse of fossil intensity.

The price of fossil energy net of transportation costs and regional taxes is assumed to be equalized across regions. But gross prices can differ depending on region-specific markups, including transportation costs and energy taxes. Over time, the fossil intensity $1/\xi_{j,t}$ falls, reducing the amount of carbon required per unit of carbon energy service. Fossil fuel is exhaustible and the cost of its production increases in cumulated historic extraction. Specifically, the model assumes that the cost of extracting fossil fuel increases sharply when cumulated extraction reaches a critical level $CumC$. Fossil production costs are thus specified as

$$q_t = \chi_1 + \chi_2 \left(\frac{\sum_{s=1995}^{t} E_s}{CumC} \right)^4,$$

where q_t is the cost of producing fossil fuel in period t and $E_t = \sum_j e_{j,t}$ is global fossil fuel use in period t. Parameters χ_1, χ_2 and $CumC$ are chosen so that supply is quite elastic initially, but very inelastic when cumulated extraction approaches $CumC$. Since agents are forward-looking, this implies that the price of fossil fuel includes a so-called Hotelling term representing the increase in future extraction costs from a marginal unit of fossil fuel use (Hotelling, 1931).

Finally, aggregate fossil fuel use E_t enters as the emission term in (14), which closes the model. Emission of fossil carbon into the atmosphere enters the carbon-circulation system, driving atmospheric carbon concentration. Through the greenhouse effect, this raises global-mean temperature T_t which reduces outputs via $\Omega_j(T_t)$, the regional damage functions. Since any emitting firm is small and the damages are spread across the whole world, the effect of the firm's emissions on its own productivity is negligible and the firm does not internalize it. However, the aggregate effect of all emissions in the world is certainly not negligible, thus creating a failure of unregulated markets.

Model calibration and welfare calculations A basic principle behind Nordhaus's modeling is that the models should be given a positive interpretation – i.e., they should be consistent with the relevant observations in reality. When it comes to the science components that describe climate and carbon circulation this is the only reasonable approach – one should chose parameters and other model characteristics to make the model reasonably consistent with historic data, and use the available theory for projections into the future.

Nordhaus argues that one should chose the details of the economic model in the same manner. In this sense, he follows the approach in modern macroeconomics, where historical data as well as microeconomic information are used to calibrate different parameters (see, e.g., Royal Swedish Academy of Sciences, 2004).

However, some economic parameters also enter into normative judgments. Take the discount rate β, which captures the relative weight placed on future generations. This parameter can be calibrated to match how much households save and bequeath.[34] This reflects a positive element of modeling. But one can also see the discount factor as a normative parameter, reflecting how much weight to put on the future in the welfare calculations. The modeler using the IAM thus has a choice. Should she base the welfare calculations on a discount rate in line with empirical observations or on ethical considerations? Here, it is important to note that Nordhaus's models can consider one discount rate when modeling individual decisions by households and another discount rate when evaluating aggregate welfare. Although the most common choice in economics is to use the same discount rate in the two cases, a strong case can be made that climate economics is different since it is concerned with much longer time perspectives than in most other applications. However, if one uses a different discount rate for aggregating welfare than the one calibrated for individual decisions, this does not only change the optimal tax rate. Typically, other policies, like subsidies to saving and investments would be called for if individuals use a higher discount rate than the one considered ethically justified when aggregating welfare.

Analogously, because gifts across (rather than within) dynasties play a quantitatively minor role in actual economies, Nordhaus assumes households in the model not to derive utility from the welfare of other households (e.g., people in other regions in the world). However, a modeler can assign any weights across regions when forming a social welfare function. One possible set of weights is the one which generates the current world distribution of consumption, but whether these are the proper welfare weights is, again, an ethical issue. Again, if more weight is put on the welfare of poor people, this has implications not only for optimal carbon taxes but also for international transfer systems.

However, some questions can be answered in economic models – including IAMs – without imposing any welfare weights. In particular, one can seek to identify policies that result in Pareto-optimal allocations. These have the property that no individual can be made better off, without at least someone made worse off. A policy that implements such an allocation, is often labelled "optimal". Intuitively, this concept maximizes the "size of the pie" but remains silent on how the pie should be divided.

In what follows, we (mostly) use the word optimal in the same way Nordhaus does – i.e., to reflect the welfare weights that govern the behavior of the model's households.

Model solution When parameters have been specified, the model must be solved. Concretely, this means finding the decision rules of the agents in the model. The approach for solving RICE and DICE differs across applications and numerical program packages, but all models are fully consistent general-equilibrium models.

As already mentioned, private agents in the economy (firms and households) do not take any externalities into account. Thus, in the market equilibrium, agents ignore the effects of their own fossil fuel use on current and future factor productivities. $\Omega_j(T_t) A_{j,t}$. Under

[34] The model's implications for interest rates is tied to the discount factor, and observations of the interest rate are thus used to measure the discount factor.

this assumption, the economic model can be solved with standard methods according to the market-equilibrium approach. For example, energy use is determined from equilibrium in the energy market, which requires the marginal product of energy derived from (17) to equal the price of fuel determined from costs, markups, and the Hotelling term. When using the market-equilibrium approach to solve the model, any optimal tax is calculated from the Pigou principle – it is set equal to the present discounted value of the effects on current and future output of a marginal unit of emissions.

The optimal solution can also be found directly as a central-planning solution. In this case, the path of all endogenous variables (consumption, capital and fuel) are chosen to maximize a weighted sum of regional welfare (16) subject to the constraints given by production functions, factor supply, carbon circulation, and the climate model. The so-called Negishi weights used to aggregated regional welfare are picked to make the allocation consistent with the actual distribution of consumption across the world's regions. The solution to the model (the path of endogenous variables) now coincides with the equilibrium allocation that would arise in a market economy at a particular set of prices. When the central planner's maximization takes full account of the effects of fossil emissions on productivity, the allocation requires fossil-fuel taxes or emission permits to be implemented as a market equilibrium. The required taxes – or, equivalently, the market prices of emission permits – follow from this maximization and are thus optimal, given the model and its parameters.

The planning solution is also conveniently used when finding the non-cooperative solution to RICE (Nordhaus and Yang, 1996). Then, one assumes that each region maximizes its own welfare given by (16), while taking the behavior of other regions as given. Since regions are relatively large, their chosen emissions do have a non-negligible impact on their own regional productivity and welfare. However, since this impact is much smaller than the global impact, emission mitigation is much too low from a global welfare perspective.

By manipulating the damage function, scenarios like limits on the amount of acceptable climate change can be studied by a maximization approach. For example, by making the damage function highly convex at a 2 °C threshold, one can find the optimal allocation conditional on remaining below the threshold. This is far from a trivial exercise, since many policies can satisfy the temperature constraint. It is then important to analyze if these policies differ in their welfare implications and, if so, find the one that fulfills the constraint at the lowest possible cost.

4.3 Putting the Model to Use

IAMs have been developed in various dimensions and we discuss these in Section 4.4 below. However, the basic structure of the latest versions of Nordhaus's own DICE and RICE models as well as models like CETA (Peck and Teisberg, 1992), MERGE (Manne et al., 1995), WITCH (Bosetti et al., 2006), and Golosov et al. (2014) is quite similar to Nordhaus's first models.[35] These models are used to assess the consequences of various policy alternatives and

[35]Another modeling approach rooted in engineering and the natural sciences also exists. Here, a more detailed description of the energy sector and the physical effects of climate change are modelled. This

to consistently simulate global warming. Nordhaus (2014) lists a number of key applications of IAMs:

- making projections with consistent inputs and outputs of the different model components
- calculating how alternative assumptions shape important variables such as output, emissions, temperature change, and damages
- tracing out the effects of alternative policies in a consistent way, and estimating the costs and benefits of alternative strategies
- estimating the uncertainties associated with alternative variables and strategies
- calculating the effects of reduced uncertainties about key parameters or variables, as well as estimating the value of research and new technologies.

In this section, we discuss some of these applications, including their quantitative contents.

Defining and analyzing scenarios Applications often call for an IAM to be run with different sets of assumptions. Different *scenarios* can, e.g., represent different assumptions about how policy will be undertaken in the future. But they can also check the sensitivity to uncertain parameters, e.g., climate and damage sensitivities. The model produces logically coherent predictions, conditional on the assumptions in different scenarios, but cannot itself produce estimates of their likelihood. Nevertheless, these conditional predictions are important, not least in policy discussion. For example, some uncertainties may be difficult – or even impossible – to quantify, given current knowledge. Then, it becomes valuable to analyze optimal policies conditional on exogenous constraints on policy, e.g., the requirement to stay below a certain amount of warming.

Because optimal policies, as defined here, find Pareto-efficient solutions based on given (sets of) welfare weights, their practical use relies on being able to compensate potential losers from policy changes. In practice, the lack of a global system for redistribution may make it politically impossible to make everyone – or even a majority – agree to the implementation of a particular policy. Thus, it is of great interest to analyze not only "first-best" policies but also second-best policies that fulfill some distributional constraints. The DICE/RICE models – and subsequently developed IAMs – are highly suitable for such analyses.

A very large number of scenarios have been analyzed over the years since Nordhaus constructed the first IAM. As an example, we present the result from a recent study (Nordhaus,

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modeling is complex and, given available computer technology and numerical techniques, not possible to combine with explicit modeling of markets with forward-looking agents in a neoclassic growth framework. These models are complementary to IAMs following the tradition of Nordhaus, by answering other types of questions – e.g., how different mitigation scenarios translate into needs for specific investments and fuel substitution. See Weyant (2017) for a recent overview.

2017). This study solves and simulates the latest vintage of the DICE model (DICE-2016) for four scenarios:

1. **Baseline**: no climate-change policies are adopted, over and above the limited policies already adopted in 2015.[36]

2. **Optimal**: paths for climate-change policies are chosen to maximize aggregate (weighted) welfare within the model from 2015 forward.

3. **Temperature-limited**: optimal policy paths are chosen, subject to the further constraint that the global temperature does not exceed 2.5 °C above the 1900 average.

4. **Stern discounting:** optimal policy paths are chosen for a subjective discount rate set to 0.1% per year, as suggested in the influential Stern Review (Stern 2007).

Due to the large uncertainty about many of the model's parameter values, Nordhaus (2017) solves the model for a large set of parameters. Figures 3 and 4 presents model outputs for median (*best-guess*) values of the parameters. Figure 3 shows the path of emissions and Figure 4 the simulated temperatures in these four scenarios.

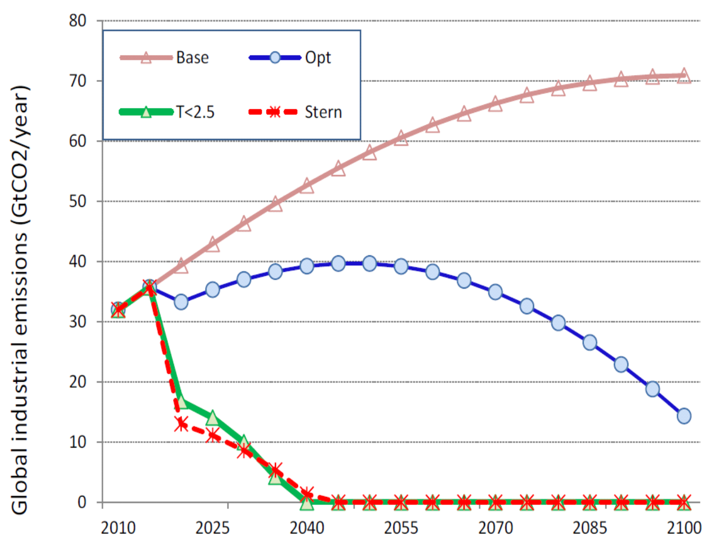

Figure 3: Emissions of CO_2 in four scenarios. Predictions from DICE-2016R2.

As Figure 3 illustrates, emissions are quite different in the four scenarios. The baseline scenario implies continuously increasing emissions. Instead, the temperature-limited and Stern discounting optimal scenarios both have radical and immediate reductions in emissions. The optimal scenario, for Nordhaus's own best guess of parameters, entails a small rise in emissions, followed by falling emissions from the middle of the century.

[36]Nordhaus estimates these policies to be equivalent of a carbon tax of USD 2/metric ton CO_2.

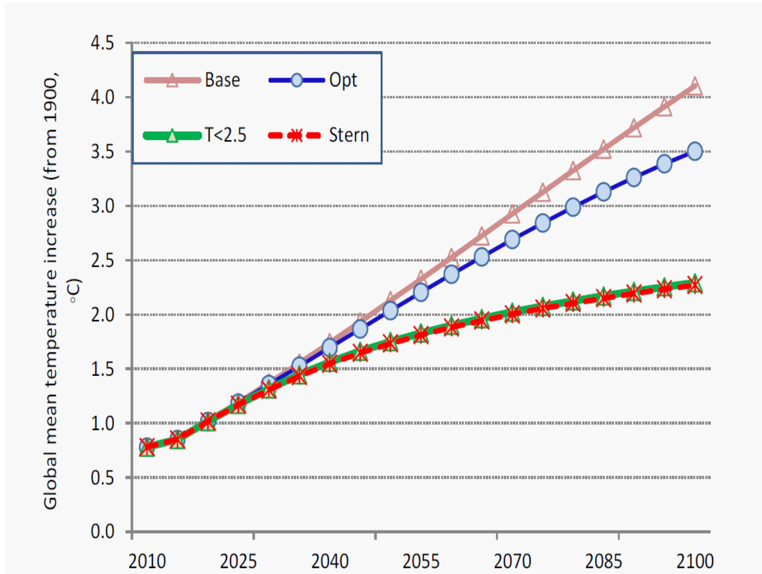

Figure 4: Increases in the global mean temperature. Predictions from DICE-2016R2.

Figure 4 shows how global mean temperature evolves in the four scenarios. Due to the inertia in the earth system, the differences are not very large until the second half of the century. Towards the end of century, however, the max-min range is close to 2°C.

The social cost of carbon Another central use of the DICE and RICE models is to calculate the *social cost of carbon*. This is defined as the present value of the damage stream resulting from a marginal unit of fossil-fuel emissions. Absent any interactions with other market failures, the social cost of carbon coincides with the optimal tax. To calculate the social cost of carbon, requires the full IAM. Specifically, (i) the carbon-cycle module is needed to predict how a unit of carbon emissions affects the path of future atmospheric CO_2 concentration, (ii) the climate module is needed to predict how a changed path of CO_2 concentration alters the climate (global temperatures), and (iii) the economic model is needed to value the economic and social damages.

In Table 1, the first row gives the social cost of carbon – i.e., the model's optimal carbon tax per metric ton CO_2, given Nordhaus's best-guess parameters. The second row provides the tax necessary to limit the warming to $2.5°C$ in the most efficient way. The third row is the optimal tax, given the low discount rate of 0.1% per year suggested in Stern (2007). Clearly, the policies are very different, the two scenarios with sharp immediate emissions reductions in Figure 3 requiring 5-10 times higher tax rates than the model's social cost of carbon. As the table shows, all three profiles rise over time – the upward slope mainly reflects the growth of GDP and real wages.

Table 1 Carbon taxes 2010 US Dollars	2015	2020	2025	2030	2050
Optimal (Nordhaus's best parameter guess)	29.5	35.3	49.1	64.0	153.5
Optimal (Temperature Limit <2.5°C)	184.1	229.0	284.0	351.0	1008.4
Optimal (Stern discounting at 0.1%)	256.5	299.6	340.7	381.7	615.6

Parameter uncertainty As discussed above, some model parameters are highly uncertain. How reliable are the predictions from the model based on the best-guess set of parameters? One way of addressing this question is to specify a distribution for the unknown parameters and solve the model for each parameter combination. This procedure is straightforward to implement in IAMs such as DICE/RICE and allows the user to assess the robustness of the model's key predictions in different dimensions.

Nordhaus (2017) provides such a sensitivity analysis, for five key uncertain parameters: (i) the coefficient on squared temperature in the damage function, (ii) the growth rate of aggregate productivity, (iii) the speed at which the economy decarbonizes through technical change, (iv) the climate sensitivity, and (v) the capacity of the intermediate carbon reservoir M^U to store carbon. For each of these, 5 quintile variables are specified, producing $5^5 = 3,125$ possible parameter combinations. The model is solved for each of these combinations, both for the optimal solution and business as usual. This produces distributions for the key variables of interest. For example, the optimal tax at the start of the simulation period (2015) is distributed with a mean of 32.5USD/ton CO_2 and a standard deviation of 28.6. Without any further climate policy, the temperature in 2100 has a mean increase of 4.2°C with a standard deviation of 1.12°C. Under the optimal tax, both the level and the variability is lower, at 3.5°C and 0.75°C, respectively. The reason for the lower variability is that parameter uncertainty is counteracted by variations in the optimal tax rate. For example, if climate sensitivity is high, this calls for higher taxes.[37]

The model's robustness features can be illustrated with box plots. Figure 5 shows such a box plot for the optimal tax. The dot is the mean over the 3,125 simulations, the box shows the middle 50% of the realizations, and the bars contain 99% of the realizations (for a normal distribution). Clearly, uncertainty about parameters leads to large policy uncertainty. This finding calls for caution and for combining predictions based on best-guess parameters with robustness analyses of this kind.[38]

Real-world policymaking How should climate policy be conducted in practice? How has it been conducted to date? We briefly discuss these issues with particular emphasis on how actual policy relates to the policies proposed in Nordhaus's research.

In RICE and DICE, a decentralized market equilibrium, where the external collective climate damages of carbon emissions balance the private benefits of carbon use, requires that

[37] An issue about learning arises. Realistically, policy needs to be decided before the uncertainty about parameters is resolved. This creates a precautionary motive for stringent policy. We discuss this issue in the next section.

[38] Gillingham et al. (2018) extend the analysis of uncertainty to cover model uncertainty, finding that parameter uncertainty is more important than model uncertainty given the existing set of models.

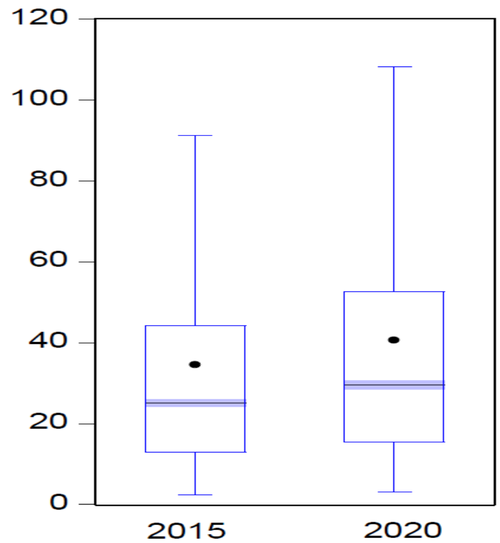

Figure 5: Box plot for optimal tax 2010$/t$CO_2$.

emitters face the right marginal cost of emitting. However, the model is silent on whether this principle should be implemented using carbon taxes or tradable emissions permits. Although there may be practical advantages of using a tax, emission-trading systems are used at least as much in practice.[39] In both cases, carbon emissions are costly for the emitter, and when the price of the permit equals the tax, the two interventions are equivalent. The World Bank produces an annual report on systems to price carbon emissions (World Bank, 2018). The data in that report shows that the number of carbon-pricing schemes is increasing over time at quite a rapid rate. The revenues from carbon-pricing schemes thus increased from USD 52 billion in 2017 to USD 82 billion in 2018 (World Bank, 2018). The carbon-pricing schemes currently in place, or scheduled to be put in place, cover about 20% of global emissions. This is far from the full global coverage that Nordhaus, and climate-economy research more generally, prescribes.

The emission trading system in the EU ("EU ETS"), is the largest among the implemented carbon-pricing systems, covering around 45% of EU emissions. A problem with this system has been a low price of emission rights, relative to the estimated social cost of carbon. Recent reforms of the system have attempted to reduce the supply of unused emission rights, and prices have since approached the optimal taxes prescribed by the best-guess version of DICE-2016. The scheduled national emission trading system in China will be of a similar size as EU ETS (World Bank, 2018).

A monetized value of the damages from carbon emissions is also useful for other domains

[39] With uncertainty, a tax gives the policy maker better control over the price of emissions while emission trading increases control over the quantity. See Weitzman (1974) for an early and general treatment of the issue of using prices or quantities as the control variable.

of policymaking. For example, the U.S. Environmental Protection Agency (EPA), in collaboration with other agencies, has produced estimates of the social cost of carbon (SCC) to be used in cost-benefit analyses of federal initiatives and regulations. According to the EPA, the purpose of producing these estimates is "to allow agencies to incorporate the social benefits of reducing carbon dioxide (CO_2) emissions into cost-benefit analyses of regulatory actions" (EPA 2016). Nordhaus's DICE model was a key input in the analysis. As discussed above, the SCC depends on the discount rate used – a parameter with an important ethical component. Therefore, the published work of EPA gave different values for the SCC for different discount rates. The latest update, from August 2016, stated the current SCC at USD 36 and 56 per ton CO_2 at discount rates of 3 and 2.5% per year, respectively.[40] The EPA also provided a quantification of the uncertainty embodied in the SCC – e.g., the 95th percentile of the SCC at 3% discounting is 105 USD/metric ton CO_2. Since 2018, however, the EPA no longer provides estimates of the SCC.

4.4 Extensions and Model Developments

Our understanding of the processes behind climate change, and how climate change affects the economy and society, is rapidly evolving. This subsection discusses a number of important extensions and model developments.

Parameter updates Nordhaus's DICE and RICE models have been easily adapted to the steady flow of new knowledge from the natural-science and economics communities. These models are open-source and available in transparent and user-friendly spreadsheet versions. It is straightforward for users to change model parameters, e.g., those in the damage function. Nordhaus himself has continuously updated his models and documented their evolution.

A very recent example is Nordhaus (2018), where several new features appear, including an explicit representation of sea-level rise – a consequence of the melting of ice sheets and thermal expansion – and its economic costs. Another important example is the re-calibration of the carbon-circulation parameters, adapted to a weaker ability of the oceans to absorb atmospheric carbon. Damage estimates, as discussed more below, have also been updated.

Together, these updates have substantially raised the estimated SCC and, thus, the optimal carbon tax. Over 25 years of revisions, the SCC in Nordhaus's models has increased from USD 5 to USD 31 per ton of CO_2. Nordhaus notes: "While this large a change is unsettling, it must be recognized that there is a large estimated error in the SCC. The estimated (5%, 95%) uncertainty band for the SCC in the 2016R model is (USD6-USD93) per ton of CO_2. This wide band reflects the compounding uncertainties of temperature sensitivity, output growth, damage function, and other factors" (Nordhaus, 2017).

We now discuss the developments underlying the parameter updates in more detail.

[40] It should be noted that these numbers represent the global damages associated with marginal emissions. EPA also provided estimates where only damages in the U.S. are included. These are an order of magnitude smaller, pointing to the large coordination problem involved in implementing globally appropriate policies.

Damage estimates The bottom-up approach underlying the damage functions in Nordhaus's IAMs has been complemented by other approaches pursued by economists. An alternative and complementary way of estimating the aggregate effects of climate on economic activity is to use reduced-form relations in data on economic outcomes and temperature. Here, both time variation and regional variation have been used to infer the effects of climate change. Regarding the former, Dell et al. (2014) summarizes a literature that studies how natural variation in temperature and climate characteristics affect economic outcomes. They conclude that, in poor countries, losses for output, labor productivity, and economic growth may be on the order of 1–2% per degree Celsius. These effects are identified using temporary changes in temperature and are arguably well identified short-run causal effects. However, the authors caution against inferring permanent effects from these estimates.

There is also a systematic relation between geographic variation in temperature and economic output. Nordhaus (2006) uses output data for 25,000 1-by-1 degree terrestrial grid cells to show a hump-shaped pattern between temperature and output. The peak of the hump, with the highest average output per square kilometer, is found at approximately 12°C.[41] Under the assumption that the cross-sectional relation between temperature and output is invariant, we can use it to infer the effects of climate change on global GDP. The estimates in Nordhaus (2006) indicate losses on the order of a few percent of global GDP if global-mean temperature increases by 3°C. While these estimates rely on strong assumptions, they do not suffer from being identified from short-run variations and are therefore complementary to time-series approaches.

The measurement of damages also involves taking into account the costs and benefits of various ways of adapting to climate change. In some cases, it is straightforward to calculate the costs of adaptation. For example, the cost of increased reliance on air conditioning can be calculated with reasonable accuracy. However, it is much more difficult to assess other forms of adaptation, such as migration. Since climate change affects different regions very differently, it will likely create large migration pressures. Migration has been a powerful adaptation mechanism helping humanity deal with historical climate changes and it has the potential to help in the future as well. At the same time, migration can lead to conflicts within and between countries, and such costs are very difficult to pin down.[42]

Thresholds and tipping points The damage function (15) as well as the climate (13) and the carbon-cycle modules (14) are smooth functions of their driving variables. The damage function (15) features limited convexity, implying that the SCC is not very sensitive to the amount of emissions. Clearly, these assumptions may prove inaccurate. For example, the damage function may become much more convex outside the range where it can be calibrated to historical data. Recent versions of RICE and DICE have included this possibility, by allowing highly convex damages when global-mean temperature reaches some critical level.

The climate and the carbon-cycle may include thresholds beyond which climate dynamics

[41]However, the (average) relation between output *per person* and temperature is monotone and negative.

[42]For studies of climate-related migration, see, e.g., Desmet and Rossi-Hansberg (2015), Feng, Krueger, and Oppenheimer (2010), and Harari and la Ferrara (2018).

change abruptly. For example, climate sensitivity can rise sharply due to stronger feedback effects in the climate system when a global temperature threshold is passed. An altered climate can change the dynamics of the carbon cycle by making reservoirs release, rather than absorb, atmospheric carbon beyond a certain point. As nonlinearities of these kinds can be many and interact with each other, the overall dynamics of the earth system becomes very hard to forecast.[43] In a series of articles, Weitzman argues (Weitzman, 2009, 2014, and Wagner and Weitzman, 2015) that low-probability catastrophic events should be the main policy concern and motivate a substantially stricter climate policy than the most likely events. Climate policy becomes an insurance policy that reduces the risk of low-probability catastrophic events for an acceptably low insurance premium. How nonlinearities in biophysical, as well as economic, systems may interact to generate thresholds and multiple steady states is also the focus of an expanding literature on the resilience of our global system (e.g., Folke, 2006, and Steffen et al., 2015). Alley et al. (2003) provide an early account of policy implications once we allow for the possibility of abrupt climate change.

A fundamental problem in this context is that the difficulty of judging the likelihood of such low-probability contingencies. It is nevertheless informative to use an IAM to assess the consequences of these contingencies, were they to occur. To this end, Nordhaus (2013) considers two extreme parameter values, one in the natural-science domain and one in the economic domain. Specifically, he considers a climate sensitivity of 10, and a damage threshold of 3°C above which the temperature coefficient in the damage function increases from 2 to 6. These values, in combination, increase the optimal carbon tax by a factor of eight. In the absence of policy, moreover, the outcome is catastrophic: the social cost of carbon in a business-as-usual scenario increases by a factor of over 100 and the economy "collapses", with flow damages rising to 96% of global GDP.

Even though science has not yet firmly concluded how much global temperatures can rise before triggering important tipping points, it is important to include such features in models that attempt to quantify the social cost of carbon and optimal policy. The DICE/RICE framework has proven capable of including abrupt changes in, e.g., the carbon-cycle and the climate modules, even if sometimes a more sophisticated solution mechanism must be used. Lemoine and Traeger (2016) extend a version of DICE by allowing tipping points in all three sub-modules of the model. Their conclusion is that these tipping points double the social cost of carbon. Lontzek, Cai, Judd, and Lenton (2016) add a number of tipping events to DICE, including an irreversible melt of the Greenland Ice Sheet, a dieback of the Amazon rainforest, and a larger amplitude of the El Niño Southern Oscillation. These tipping events are stochastic, and their probability is assumed to increase in global-mean temperature. Furthermore, the impact is allowed to accumulate slowly, but irreversibly. The inclusion of these events raises the SCC by around 50%.[44]

[43] See Lenton et al. (2008) for an overview.

[44] There is now a fairly large literature considering thresholds and tipping points in IAM's. Examples include Gjerde, Grepperud, and Kverndokk (1999), Castelnuovo, Moretto, and Vergalli (2003), and van der Ploeg and de Zeeuw (2015).

Discounting The Stern Review (2007) was commissioned by U.K. finance minister Gordon Brown to provide guidelines in climate policy. The review had enormous impact in policy circles and one of its key inputs was Nordhaus's work, though the review also used other assessment models and imposed its own different parameter choices. In particular, the Stern Review argues that it is inappropriate to discount the welfare of future generations and calls for a very low discount rate to be used when calculating the SCC. The report uses a discount rate for welfare of 0.1%, per year in contrast to DICE/RICE, which assumes a discount rate at around 1.5%.

As pointed out above, one can use an ethical argument against (high) discount rates on future welfare. On the other hand – and more in line with Nordhaus's approach – when market prices are used to infer how households themselves discount future welfare, one typically finds a welfare discount rate of a percent or so per year. Furthermore, market returns provide information on alternative ways of transferring resources to the future. If current generations want to engage in such transfers, one could argue that they should do so in the most efficient way – mitigating climate change is only one possibility. Nevertheless, which welfare discount rates to use has always been discussed in cost-benefit analyses of large infrastructure projects. These issues are highly salient and of large quantitative importance in the area of climate economics, where the time horizon is particularly long.

Since carbon emitted into the atmosphere remains there for a very long time (a substantial share remains after thousands of years), current emissions can cause damages very far out in the future. A discount rate of 0.1% implies that the weight on welfare 500 years from now is 0.60. A discount rate of 1.5% per year instead implies a weight of 0.0005. Given this, it is not difficult to understand that the lower discount rate produces a much higher social cost of carbon, all else equal. To illustrate this point within the IAM framework, assume that the damage per unit of excess carbon in the atmosphere is a constant share of GDP (which is approximately the case in DICE/RICE), the utility of consumption is logarithmic, and the saving rate is constant. Then, it is straightforward to show that the SCC is proportional to the carbon duration with welfare discount rate weights, defined as

$$D(\rho) = \int_0^\infty (1 - d_s) e^{-\rho s} ds,$$

where $1 - d_s$ is the share of carbon that remains in the atmosphere s periods after emission and $\rho = 1 - \beta$ is the welfare discount rate.[45] Figure 7 shows the function $D(\rho)$ for standard assumptions about carbon depreciation from IPCC (2007).[46]

[45] Generally, the social cost of carbon in period t is

$$\int_0^\infty e^{-\rho s} \frac{u_c(c_{t+s})}{u_c(c_t)} \frac{\partial Y_{t+s}}{\partial S_{t+s}} \frac{\partial Y_{t+s}}{\partial E_t} ds,$$

with logarithmic utility, an (approximately) constant savings rate, and under the assumption $\frac{\partial Y_{t+s}}{\partial S_{t+s}} = \gamma Y_{t+s}$ this reduces to $\gamma Y_t D(\rho)$.

[46] $1 - d(s) = a_0 + \sum_{i=1}^3 \left(a_i e^{-\frac{s}{\tau_i}}\right)$, with $a_0 = 0.217$, $a_1 = 0.259$, $a_2 = 0.338$, $a_3 = 0.186$, $\tau_1 = 172.9$, $\tau_2 = 18.51$, and $\tau_3 = 1.186$, for s measured in years.

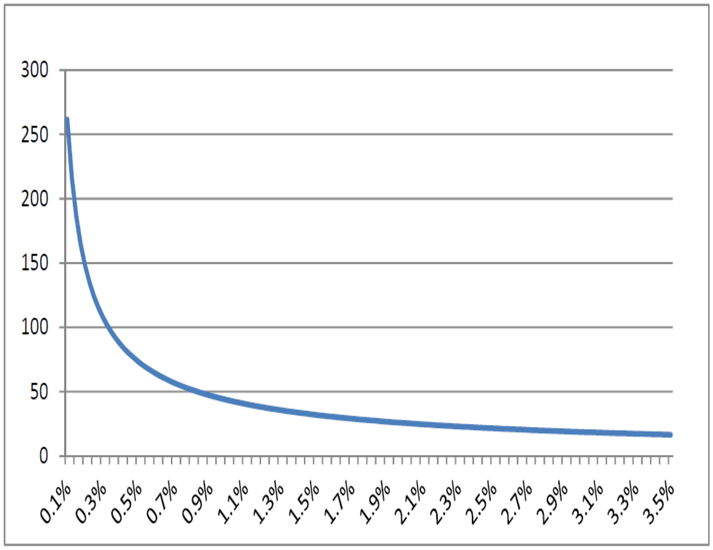

Figure 6: Proportionality factor in the formula for the optimal tax with logarithmic utility and constant saving.

All else equal, the social cost of carbon is 8.2 times higher at a utility discount rate of 0.1% rather than 1.5% per year. Thus, IAMs suggest a very high sensitivity of the SCC to the discount rate. This is an important result. The "all else equal" is an important qualification, however. If we still require that the model generate realistic capital returns, a reduction of the welfare discount rate requires a reduction in the intertemporal elasticity of substitution. Then, the effect of reducing the welfare discount rate on the optimal tax is largely muted (Nordhaus, 2014).

Another possibility is that – on normative grounds – we genuinely believe that people on average put too little weight on the future and therefore save (and invest) too little. Then, the optimal path of carbon taxes calculated at a low discount rate will have to be accompanied by a (global) subsidy to savings in order to implement the equilibrium solution associated with these taxes.

Technology Technological change is of first-order importance for the economy's reaction to climate change and for the policy instruments used to deal with it. The original RICE/DICE models are built on the Solow exogenous growth model, but later developments have followed the spirit of Romer and endogenized technological improvements. A prominent example is the WITCH-model (World Induced Technical Change Hybrid Model, Bosetti et al., 2006). This starts out from the basic structure of RICE, but adds a more detailed description of the energy sector and the possibility of carbon sequestration. Technological change that reduces the cost of supplying energy, both fossil and green, is modelled according to two endogenous processes. One process is learning-by-doing, where the production cost depends on the global installed capacity. As in Romer (1986), this creates an externality and too slow adoption of new technologies in an unregulated economy. The other process is purposeful R&D as in Romer (1990) directed into energy efficiency and the production of biofuel, which also creates a spillover: ideas created by R&D in one region, can be used in other regions albeit with a lag. Other examples of models building on RICE/DICE with endogenous technological change are those based on directed technical change, and mentioned already in Section 3.4.

Political economy The standard IAM considers policy as an exogenous choice: it has no "political-economy" elements. Yet, implementation constraints can be crucial for such policies. One reason is that the policy scope is global and thus involves many regions and countries. Related to this global compensation schemes may be necessary to implement optimal carbon pricing. Yet another reason is that the very long policy horizon may collide with short-run political horizons.

IAMs can still be useful for positive analyses of policy. For example, Nordhaus and Yang (1996) use the DICE model to compare the optimal fossil carbon tax to the tax that would result in a Nash equilibrium without international cooperation. Their results suggest that global cooperation is of key importance. Without cooperation, equilibrium policy would entail an average tax rate, which is only 1/25th of the global optimal tax. The political-economy literature on climate change is an active and important new research area. For an early survey, see Kolstad and Toman (2005), and for a recent contribution to the theoretical literature on climate agreements, see Battaglini and Harstad (2016).

5 Concluding Remarks

Paul M. Romer and William D. Nordhaus have devised new tools for analyzing long-run development. From a long-run global perspective, technological change and climate change are key aspects of sustained and sustainable long-run economic growth. The two scholars have been highly influential within a broad research community. Both of them have taken the same starting point, namely the neoclassical growth model, and amended it with key drivers of long-run economic activity – technological developments and climate – that had been highlighted by economic historians, but treated as exogenous by most economists. Both of them have emphasized externalities in their analysis of desirable long-run outcomes, thus pointing to a potentially important role for economic policy and offering new guidance for its design.

Looking forward, the combined work by the Laureates offers the research community an opportunity to address long-run issues around climate, energy supply and sustainability, by studying government policy together with endogenous technological change in the global market economy.

The Royal Swedish Academy of Sciences has decided to award the Sveriges Riksbank Prize in Economic Sciences for 2018 to be shared equallly between William D. Nordhaus, Yale University,
for integrating climate change into long-run macroeconomic analysis

and Paul M. Romer, New York University,
for integrating technological innovations into long-run macroeconomic analysis.

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